

Gluon self-interaction in the position space in Landau gauge

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Abstract

We propose a method to treat the three-gluon self-interaction vertex in the position space in $D = 4 - 2\epsilon$ dimensions. As an example, we calculate a two-loop contribution to auxiliary Lcc vertex in the Landau gauge which contains the three-gluon vertex for $SU(N)$ Yang-Mills theory. We represent the integral expression as a sum of separate contributions so that each of the contributions is a double finite integral or single integral (singular or finite) in the position space. In each double finite integral we use the freedom to shift exponents in powers in the denominator of integrands by some multiples of ϵ , in order to perform at least one of the integrations by the uniqueness technique without corrupting the first term of the decomposition in ϵ .

Keywords: Lcc vertex, Gegenbauer polynomial technique, Davydychev integral $J(1, 1, 1)$

1 Introduction

Transversality of the vector propagator in the Landau gauge causes the existence of the finite scalar auxiliary three-point vertex Lcc in the Wess-Zumino gauge of $\mathcal{N} = 4$ supersymmetric Yang-Mills theory. As it has been shown in Refs. [1]-[5], all the poles in ϵ disappear in all number of loops for that vertex. This result has been derived from the results of Refs. [6]-[10]. This vertex does not depend on any scale, ultraviolet (UV) or infrared (IR), to all orders of the perturbation theory for maximally supersymmetric Yang-Mills theory. Parameter ϵ is the parameter of dimensional regularization, $D = 4 - 2\epsilon$ is the dimension of the space-time. The first two vertex-like two-loop contributions that correspond to diagrams (a) and (b) were calculated in the previous papers, Refs. [4, 5], respectively. In this work we calculate the contribution that corresponds to diagram (c). The notation that is used here is the notation of Ref. [4].

The Lcc vertex is the vertex in which the auxiliary field L couples to two (self-adjoint) Faddeev-Popov ghost fields c . It is superficially convergent in the Landau gauge. This fact can be checked by index counting and by noting that two derivatives from the ghost propagators can always be integrated out of the diagram due to the transversality of the gauge propagator. It means that the field c does not have renormalization in the Landau gauge. Formally, this result holds to all orders of perturbation theory due to the so called antighost equation [11]. In the nonsupersymmetric theories this vertex is not finite and a calculation of the anomalous dimension of operator cc has been performed in [12, 13].

In Refs. [1, 2, 3] it has been shown that if the vertex Lcc is known to all loop orders of the maximally supersymmetric Yang-Mills theory, one can obtain all the other correlators of dressed mean fields by solving the Slavnov-Taylor identity. However, despite all-order UV and IR finiteness of this scalar vertex, its all-order calculation presents in itself an outstanding task for an analytic programmer. At this stage, we can present only full planar two-loop result in the Landau gauge. The difficult part is the algebra of Lorentz indices in the gluon self-interaction part. We have solved this problem at two-loop level by doing algebra of convolution of the derivatives and integrating by parts. This trick allows us to keep the extensive Lorentz algebra under control and to use, at the subsequent stage, *Mathematica* software [14]. For convenience, we present in this manuscript details of calculation up to a certain stage.

Dressed mean fields are the usual effective fields redefined by integral convolutions with parts of propagators [1, 2, 3]. The difference with the usual BPHZ renormalization procedure [15] is that these parts of propagators contain not only poles in ϵ but also space-dependent logarithmic part (or momentum-dependent logarithmic part if one works in the momentum space) constructed typically in the \overline{MS} scheme procedure [16]. This is why integral convolution was used instead of traditional simple multiplication. All poles in ϵ in Wess-Zumino gauge are contained in these dressing functions of the effective fields. Physical meaning have the kernels of those dressed effective fields in the effective action. On-shell values in the momentum space correspond to the amplitudes of physical particles, for example gluons.

As it has been shown in Refs. [1, 2, 3], scale-independence of the correlators of dressed mean fields corresponding to physical particles is a direct consequence of the Slavnov-Taylor identity [17, 18, 19, 20, 21, 22]. The latter is a consequence of the BRST symmetry [23, 24]. The scale independence of the kernels of dressed mean fields and the vanishing of the beta function of the coupling in the maximally supersymmetric Yang-Mills theory at any number of loops suggest

that the correlators of the dressed mean fields can be analyzed by the methods of conformal field theory in the position space, since the vanishing of the beta function is a consequence of the conformal symmetry of $\mathcal{N} = 4$ supersymmetric Yang–Mills theory.

In the momentum space, by using unitarity methods, it has been demonstrated that only the diagrams created due to the “rung rule” contribute in the four-point gluon amplitude up to three-loop level [25]. This “rung rule,” combined with conformal invariance of the ladder diagrams of Refs. [26, 27, 28] in the momentum space, allowed to classify all conformally invariant contributions in the momentum space up to four-loop level [31, 29, 30]. The same conformal symmetry appears in Alday-Maldacena approach [32] to calculate gluon scattering amplitude on the string side at strong coupling. Conformal invariant contributions to four-point gluon amplitude reproduce the known results for the anomalous dimensions of twist-two operators in maximally supersymmetric Yang-Mills theory [33, 34, 35, 36, 37]. Even if structure of amplitudes at the weak coupling can be found by using iterative formula of Ref. [25], the structure of off-shell correlators is still unknown. For some sub-class of diagrams it can be found exactly [38]. The Slavnov-Taylor identity can be used as an alternative method for study the full structure of the off-shell correlators.

This manuscript is organized as follows. In Sec. 2 the gluon self-interaction vertex of diagram (c) is considered. In Sec. 3 the integral structure and most important details of calculation are presented. In Subsec. 3.1 the basic ideas of the method are presented in more detail. The result consists of three parts. Each part corresponds to a particular position of the derivative of the gluon vertex. In Sec. 4 the total result for the diagram is written. Appendices A, B, C and D contain further technical details.

2 Diagram (c)

The planar two-loop correction to Lcc vertex can be represented as a combination of five diagrams depicted in Fig. 1. We introduce, for brevity, the notation

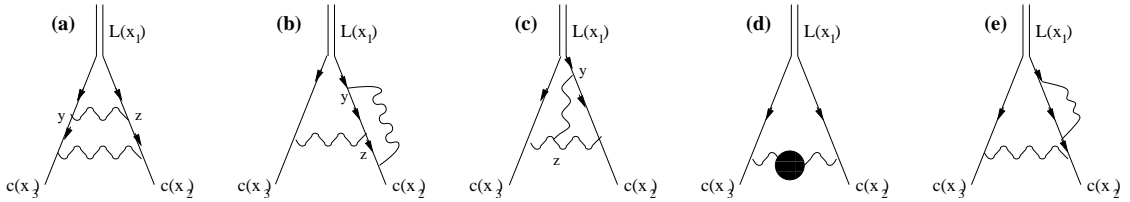


Figure 1: The two-loop diagrams for the Lcc vertex. The wavy lines represent the gluons, the straight lines the ghosts. Black disk stands for the total one-loop correction to the gluon propagator.

$$[yz] = (y - z)^2, \quad [y1] = (y - x_1)^2, \dots$$

and so on. We use the notation $\Pi_{\mu\nu}^{ab}(xy)$ for the gluon propagator from point x to point y . In the exact four-dimesional $D = 4$ space it is

$$\Pi_{\mu\nu}^{ab}(xy) = \left(\frac{g_{\mu\nu}}{[xy]} + 2 \frac{(xy)_\mu (xy)_\nu}{[xy]^2} \right) \delta^{ab} .$$

It satisfies the condition of transversality in $D = 4$. From the Yang-Mills Lagrangian we have the following three-gluon vertex which is in the center of the diagram c .

$$\int Dz \left(\partial_\mu A_\rho^A(z) \right) A_\mu^B(z) A_\rho^C(z) f^{ABC} .$$

We work with $SU(N)$ group where the structure constants are completely antisymmetric in indices. This vertex creates the interaction for the diagram. In the rest of the paper we use three triples of indices (y, λ, a) , $(2, \sigma, b)$, and $(3, \nu, c)$. Thus, for three-point gluon function we have the following contribution, which is in the center of the diagram c :

$$\begin{aligned} & \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\lambda}^{Aa}(zy) \right) \Pi_{\mu\sigma}^{Bb}(z2) \Pi_{\rho\nu}^{Cc}(z3) f^{ABC} + \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\lambda}^{Aa}(zy) \right) \Pi_{\mu\nu}^{Bc}(z3) \Pi_{\rho\sigma}^{Cb}(z2) f^{ABC} \\ & + \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}^{Ab}(z2) \right) \Pi_{\mu\lambda}^{Ba}(zy) \Pi_{\rho\nu}^{Cc}(z3) f^{ABC} + \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}^{Ab}(z2) \right) \Pi_{\mu\nu}^{Bc}(z3) \Pi_{\rho\lambda}^{Ca}(zy) f^{ABC} \\ & + \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}^{Ac}(z3) \right) \Pi_{\mu\lambda}^{Ba}(zy) \Pi_{\rho\sigma}^{Cb}(z2) f^{ABC} + \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}^{Ac}(z3) \right) \Pi_{\mu\sigma}^{Bb}(z2) \Pi_{\rho\lambda}^{Ca}(zy) f^{ABC} . \end{aligned}$$

Taking into account the antisymmetry of the indices, we obtain

$$\begin{aligned} & \int Dz \left[\left(\partial_\mu^{(z)} \Pi_{\rho\lambda}(zy) \right) \Pi_{\mu\sigma}(z2) \Pi_{\rho\nu}(z3) - \left(\partial_\mu^{(z)} \Pi_{\rho\lambda}(zy) \right) \Pi_{\mu\nu}(z3) \Pi_{\rho\sigma}(z2) \right] f^{abc} \\ & + \int Dz \left[- \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\lambda}(zy) \Pi_{\rho\nu}(z3) + \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \Pi_{\rho\lambda}(zy) \right] f^{abc} \\ & + \int Dz \left[\left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\lambda}(zy) \Pi_{\rho\sigma}(z2) - \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \Pi_{\rho\lambda}(zy) \right] f^{abc} . \end{aligned}$$

We can re-organize the expression to the following form:

$$\begin{aligned} & \int Dz \left[\left(\partial_\mu^{(z)} \Pi_{\rho\lambda}(zy) \right) \Pi_{\mu\sigma}(z2) \Pi_{\rho\nu}(z3) - \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \Pi_{\rho\lambda}(zy) \right] f^{abc} \\ & + \int Dz \left[- \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\lambda}(zy) \Pi_{\rho\nu}(z3) + \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\lambda}(zy) \Pi_{\rho\sigma}(z2) \right] f^{abc} \\ & + \int Dz \left[\left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \Pi_{\rho\lambda}(zy) - \left(\partial_\mu^{(z)} \Pi_{\rho\lambda}(zy) \right) \Pi_{\mu\nu}(z3) \Pi_{\rho\sigma}(z2) \right] f^{abc} = \\ & = \int Dz \left[\left(\partial_\mu^{(z)} \Pi_{\rho\lambda}(zy) \right) \Pi_{\rho\nu}(z3) - \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\lambda}(zy) \right] \Pi_{\mu\sigma}(z2) f^{abc} \\ & + \int Dz \left[- \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\rho\nu}(z3) + \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\sigma}(z2) \right] \Pi_{\mu\lambda}(zy) f^{abc} \\ & + \int Dz \left[\left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\rho\lambda}(zy) - \left(\partial_\mu^{(z)} \Pi_{\rho\lambda}(zy) \right) \Pi_{\rho\sigma}(z2) \right] \Pi_{\mu\nu}(z3) f^{abc} . \end{aligned} \tag{1}$$

3 Integral structure

The idea of the calculation is simple. The ghost propagator differs slightly from Dirac δ -function. Actually, one derivative convoluted with the ghost propagator immediately produces the δ -function in the position space. Each vector propagator at least once has such a convolution with the ghost propagator. We can apply the integration by parts (IBP) for such a convolution of two derivatives. The convolution can be represented as a difference of d’Alambertians. After this, the δ -functions appear. They will remove one of the integrations.

Our purpose is to reduce Lorentz algebra in the integrands in order to obtain *finite* double integrals with less number of indices, or single integrals that can be finite or singular. We use the following terminology. “Finite integrals” means that they have smooth limit in ϵ , “singular integrals” means that they have poles in ϵ . In paper [5] devoted to diagram (b) we performed one of the integrations first by the uniqueness technique and then carried out the Lorentz algebra to scalarize the second single integration. Such an approach was justified there, since the number of Lorentz indices for the diagram with two vector propagators was relatively small. In the diagram (c) there is one three-gluon vertex. Therefore, the number of Lorentz indices and the number of resulting double integrals are huge, and to keep the indices under control we apply IBP to reduce their number.

We note by index counting that all three lines of Eq. (1) are finite separately. No IR or UV poles should arise. UV behavior can be checked in the momentum space while IR behavior can be checked in the position space. In the UV region all subgraphs corresponding to each separate line of Eq. (1) are finite (it does not matter where the derivative of the three-gluon vertex stands precisely).

3.1 The first line of Eq. (1)

In this Subsection we analyze the first line of Eq. (1). We have for the first line the following expression:

$$T_1 \equiv \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma (1y)_\lambda}{[2y]^2 [1y]^2} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\lambda}(zy) \Pi_{\mu\sigma}(z2) . \quad (2)$$

The integration here is performed in $D = 4 - 2\epsilon$ dimensions. All the notations are taken from Ref. [4]. We use simple algebra to represent integral T_1 as a combination of terms with simpler Lorentz structure,

$$\begin{aligned} \Pi_{\rho\lambda}(zy) \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} &= \left(\frac{g_{\rho\lambda}}{[yz]^{1-\epsilon}} + 2(1-\epsilon) \frac{(yz)_\rho (yz)_\lambda}{[yz]^{2-\epsilon}} \right) \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} = \\ \left(\frac{2g_{\rho\lambda}}{[yz]^{1-\epsilon}} - \partial_\lambda^{(y)} \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \right) \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} &= \frac{2(1y)_\rho}{[yz]^{1-\epsilon} [1y]^{2-\epsilon}} - \left(\partial_\lambda^{(y)} \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \right) \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} = \\ \frac{2(1y)_\rho}{[yz]^{1-\epsilon} [1y]^{2-\epsilon}} - \frac{1}{2(1-\epsilon)} \left(\partial_\lambda^{(y)} \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \right) \left(\partial_\lambda^{(y)} \frac{1}{[1y]^{1-\epsilon}} \right) &= \\ \frac{2(1y)_\rho}{[yz]^{1-\epsilon} [1y]^{2-\epsilon}} - \frac{1}{4(1-\epsilon)} \left[\partial_{(y)}^2 \left(\frac{(yz)_\rho}{[yz]^{1-\epsilon} [1y]^{1-\epsilon}} \right) - \left(\partial_{(y)}^2 \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \right. \\ &\quad \left. - \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \left(\partial_{(y)}^2 \frac{1}{[1y]^{1-\epsilon}} \right) \right] . \end{aligned} \quad (3)$$

Now we use the formula of Ref. [31]¹

$$\partial_{(y)}^2 \frac{1}{[1y]^{1-\epsilon}} = k(\epsilon) \delta^{(4-2\epsilon)}(1y) ,$$

where k is some coefficient that we do not specify at this moment. Then, integral T_1 takes the form

$$\begin{aligned} T_1 &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\lambda}(zy) \Pi_{\mu\sigma}(z2) = \\ &\quad \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \times \\ &\quad \times \left\{ \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \right. \\ &\quad \left. - \frac{1}{4(1-\epsilon)} \left[\partial_{(y)}^2 \left(\frac{(yz)_\rho}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) - \left(\partial_{(y)}^2 \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} - \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \left(\partial_{(y)}^2 \frac{1}{[1y]^{1-\epsilon}} \right) \right] \right\} = \\ &\quad \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \times \\ &\quad \times \left\{ \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} + \frac{1}{4(1-\epsilon)} \left(\partial_{(y)}^2 \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \right\} - \\ &\quad - \frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \partial_{(y)}^2 \frac{(yz)_\rho}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} + \\ &\quad + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(yz)_\rho}{[yz]^{1-\epsilon}} \delta^{(4-2\epsilon)}(1y) = \\ &\quad \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \left\{ \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\rho}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\ &\quad + \frac{1}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \int Dy \frac{1}{[2y]^{1-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \partial_{(y)}^2 \frac{(yz)_\rho}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} + \\ &\quad + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(1z)_\rho}{[1z]^{1-\epsilon}} = \\ &\quad \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \left\{ \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\rho}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\ &\quad + \frac{k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \int Dy \delta(2y) \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(yz)_\rho}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} + \\ &\quad + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(1z)_\rho}{[1z]^{1-\epsilon}} = \\ &\quad \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \left\{ \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\rho}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\ &\quad + \frac{k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} + \end{aligned}$$

¹We have taken this formula from the first version of Ref. [31].

$$\begin{aligned}
& + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^{2-\epsilon}} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(1z)_\rho}{[1z]^{1-\epsilon}} \\
& \equiv T_{11} + T_{12} + T_{13} . \quad (4)
\end{aligned}$$

It is simpler to investigate singularities of single integrals than singularities of double integrals. Integrals T_{12} and T_{13} are finite. Since T_1 is finite, it means that T_{11} is also finite. Moreover, the finiteness (UV and IR) of T_{11} can be checked by index counting directly. UV-finiteness in the momentum space can be checked for the Fourier transform of integral (4) for each subgraph and for both momentum integrations together, according to BPHZ R -operation. IR-finiteness of the double integrals can be checked directly for integral (4), for each integration separately and for both integrations together in the area $|y| \rightarrow \infty$, $|z| \rightarrow \infty$, in complete analogy with R -operation of the momentum space.

$$\begin{aligned}
T_{11} = & -\frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \Pi_{\mu\sigma}(z2) \times \\
& \times \left\{ \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) - \frac{1}{[1y]^{1-\epsilon}} \left(\partial_\mu^{(z)} \frac{(yz)_\rho}{[yz]^{2-\epsilon}} \right) \right\} \equiv T_{111} + T_{112} .
\end{aligned}$$

We calculate T_{111} in detail in the main body of the paper, and the calculation of T_{112} can be found in Appendix A. The technique of calculation used in this paper is described in Ref. [4]. It is based on the uniqueness method [39, 16, 40, 41], and Gegenbauer polynomial technique introduced in Refs. [42, 43, 44, 45, 46, 47] and further developed in Ref. [48]. All the results are obtained in terms of the Davydychev integral $J(1, 1, 1)$ explicitly found in Ref. [49], and logarithms of ratios of the space-time differences of the coordinates of the effective fields in the position space. New integral representation for the Davydychev integral has been found in Ref. [4].

3.1.1 Calculation of T_{111}

$$-T_{111} \equiv \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \Pi_{\mu\sigma}(z2) \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) .$$

Again, we apply simple algebra to reduce the number of indices further

$$\begin{aligned}
\Pi_{\mu\sigma}(2z) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) &= \left(\frac{g_{\mu\sigma}}{[2z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(2z)_\mu (2z)_\sigma}{[2z]^{2-\epsilon}} \right) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
&= \left(\frac{2g_{\mu\sigma}}{[2z]^{1-\epsilon}} - \partial_\mu^{(2)} \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \right) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
&= \frac{4(1-\epsilon)(yz)_\sigma}{[2z]^{1-\epsilon}[yz]^{2-\epsilon}} + \left(\partial_\mu^{(z)} \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \right) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
&= \frac{4(1-\epsilon)(yz)_\sigma}{[2z]^{1-\epsilon}[yz]^{2-\epsilon}} + \frac{1}{2} \left[\partial_{(z)}^2 \left(\frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \right) - \left(\partial_{(z)}^2 \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} \right. \\
&\quad \left. - \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \right] . \quad (5)
\end{aligned}$$

The first contribution to $-T_{111}$ (with factor $2(1-\epsilon)$) is

$$\begin{aligned}
J_1 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{2(yz)_\sigma}{[2z]^{1-\epsilon}[yz]^{2-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
&= -\frac{1}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\sigma^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \left(\partial_\sigma^{(y)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
&= \frac{1}{4(1-\epsilon)^3} \frac{(31)_\nu}{[31]^2} \partial_\rho^{(1)} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[yz]^{1-\epsilon}[2z]^{1-\epsilon}} k\delta(1y) \\
&\quad + \frac{1}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}} k\delta(yz) \frac{1}{[2z]^{1-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} \\
&\quad + \frac{1}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) k\delta(2y) \frac{1}{[yz]^{1-\epsilon}} \frac{1}{[2z]^{1-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
&= \frac{k}{4(1-\epsilon)^3} \frac{(31)_\nu}{[31]^2} \partial_\rho^{(1)} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[21]^{1-\epsilon}[1z]^{1-\epsilon}[2z]^{1-\epsilon}} \\
&\quad + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2-\epsilon}} \\
&\quad + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} = \\
&\quad \frac{k}{4} \frac{(31)_\nu}{[31]^2} \partial_\rho^{(1)} \frac{1}{[21]} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[1z][2z]} \\
&\quad + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2-\epsilon}} \\
&\quad + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \equiv \\
&\equiv \frac{k}{4} J_{12} + \frac{k}{4(1-\epsilon)^2} M_{11} + \frac{k}{4(1-\epsilon)^2} M_{10} .
\end{aligned}$$

Integral J_1 is singular in UV. Integral J_{12} is finite in UV and IR. Integrals M_{11} and M_{10} are singular. The second contribution (with factor is $1/2$) to $-T_{111}$ is

$$\begin{aligned}
J_2 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \partial_{(z)}^2 \left(\frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \right) \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
&= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\frac{2g_{\rho\nu}}{[3z]^{1-\epsilon}} - \partial_\nu^{(3)} \frac{(3z)_\rho}{[3z]^{1-\epsilon}} \right) \partial_{(z)}^2 \left(\frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \right) \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
&= \frac{2}{1-\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \partial_\rho^{(1)} \partial_\rho^{(3)} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}[1y]^{1-\epsilon}[3z]^{1-\epsilon}} \\
&\quad - \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz 2k\delta(3z) \frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}[1y]^{1-\epsilon}} = \\
&\quad 2 \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \partial_\rho^{(1)} \partial_\rho^{(3)} \int Dy \frac{(2y)_\sigma}{[2y]^{2-2\epsilon}} \int Dz \frac{(2z)_\sigma}{[2z]^{1-2\epsilon}[yz][1y][3z]} \\
&\quad - 2k \frac{(31)_\nu}{[31]^2} \frac{(23)_\sigma}{[23]^{1-\epsilon}} \partial_\nu^{(1)} \int Dy \frac{(2y)_\sigma}{[2y]^2[y3][1y]} \equiv 2J_{21} - 2kJ_{22} .
\end{aligned}$$

We shifted exponents in powers by $\sim \epsilon$ since J_2 , J_{21} and J_{22} are finite in IR and UV. We always apply this trick to finite integrals when we need to use the uniqueness relation at least for one of the integrations (this is the trick of Ref. [4]). The third contribution (with factor $-1/2$) to $-T_{111}$ is

$$\begin{aligned}
J_3 \equiv & \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_{(z)}^2 \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
& 2 \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\sigma^{(2)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
& -\frac{1}{1-\epsilon} \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\sigma^{(2)} \frac{1}{[2y]^{1-\epsilon}} \right) \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\sigma^{(2)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
& -\frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[2y]^{1-\epsilon} [2z]^{1-\epsilon} [yz]^{1-\epsilon} [1y]^{2-\epsilon}} \\
& + \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) k\delta(2y) \frac{2(1y)_\rho}{[2z]^{1-\epsilon} [yz]^{1-\epsilon} [1y]^{2-\epsilon}} \\
& + \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) k\delta(2z) \frac{2(1y)_\rho}{[2y]^{1-\epsilon} [yz]^{1-\epsilon} [1y]^{2-\epsilon}} = \\
& -\frac{1}{2} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[2y][2z][yz][1y]^2} \\
& + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \\
& + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{2(1y)_\rho}{[2y]^{2-2\epsilon} [1y]^{2-\epsilon}} \equiv \\
& \equiv -\frac{1}{2} J_{31} + \frac{k}{2(1-\epsilon)} M_{10} + \frac{k}{2(1-\epsilon)} M_{12} .
\end{aligned}$$

Integral J_3 is divergent in UV. Integral J_{31} is finite, integrals M_{10} and M_{12} are singular. The fourth contribution (with factor $-1/2$) to $-T_{111}$

$$\begin{aligned}
J_4 \equiv & \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
& \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} k\delta(yz) \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \int Dz \frac{(2z)_\sigma}{[2z]^{2-\epsilon}} \Pi_{\rho\nu}(z3) \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \frac{2(1z)_\rho}{[1z]^{2-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \int Dz \frac{1}{[2z]^{2-2\epsilon}} \Pi_{\rho\nu}(z3) \frac{2(1z)_\rho}{[1z]^{2-\epsilon}} = k M_{11} .
\end{aligned}$$

This single integral is singular. We collect and calculate all the singular integrals from J_1 , J_3 and J_4 together in order to demonstrate that poles finally disappear in the sum of all the singular integrals. The sum of all those singular integrals is

$$M_1 \equiv \frac{k}{2(1-\epsilon)} M_{11} + \frac{k}{2(1-\epsilon)} M_{10} - \frac{k}{4(1-\epsilon)} M_{10} - \frac{k}{4(1-\epsilon)} M_{12} - \frac{k}{2} M_{11} .$$

For the beginning, we do the calculation of the first integral

$$\frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2-\epsilon}}.$$

The shift of exponents in powers $1/[1z]^{2-\epsilon} \mapsto 1/[1z]^{2+\epsilon}$ will be performed in the last denominator of the integrand in order to use the uniqueness method for the case of three factors in the denominators. Moreover, this factor $1/[1z]^{2-\epsilon}$ is present in all other integrands. In other singular integrands we will shift that exponent too. The sum of all singular integrals must be finite. The shift of exponents in powers must be the same in all divergent integrands.

$$\begin{aligned} M_{11} &= \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2-\epsilon}} \rightarrow \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2+\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dz \left(\frac{g_{\rho\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\rho(3z)_\nu}{[3z]^{2-\epsilon}} \right) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2+\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dz \left(\frac{2g_{\rho\nu}}{[3z]^{1-\epsilon}} - \partial_\nu^{(3)} \frac{(3z)_\rho}{[3z]^{1-\epsilon}} \right) \frac{1}{[2z]^{2-2\epsilon}} \frac{2(1z)_\rho}{[1z]^{2+\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dz \frac{4(1z)_\nu}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}[1z]^{2+\epsilon}} - \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dz \frac{2(3z)_\rho(1z)_\rho}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}[1z]^{2+\epsilon}} = \\ &= -\frac{2}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} \int Dz \frac{1}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}[1z]^{1+\epsilon}} - \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dz \frac{2(3z)_\rho(1z)_\rho}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}[1z]^{2+\epsilon}} = \\ &= -\frac{2}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} J(1+\epsilon, 2-2\epsilon, 1-\epsilon) - \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dz \frac{2(31)_\rho(1z)_\rho + 2(1z)_\rho(1z)_\rho}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}[1z]^{2+\epsilon}} = \\ &= -\frac{2}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} J(1+\epsilon, 2-2\epsilon, 1-\epsilon) + \frac{1}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} (31)_\rho \partial_\rho^{(1)} \int Dz \frac{1}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}[1z]^{1+\epsilon}} \\ &= -2 \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} J(1+\epsilon, 2-2\epsilon, 1-\epsilon) = \\ &= -\frac{2}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} J(1+\epsilon, 2-2\epsilon, 1-\epsilon) + \frac{1}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} (31)_\rho \partial_\rho^{(1)} J(1+\epsilon, 2-2\epsilon, 1-\epsilon) \\ &= -2 \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} J(1+\epsilon, 2-2\epsilon, 1-\epsilon) = \\ &= \left[-\frac{2}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} + \frac{1}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} (31)_\rho \partial_\rho^{(1)} - 2 \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \right] \frac{A(1+\epsilon, 2-2\epsilon, 1-\epsilon)}{[12][23]^{1-2\epsilon}[31]^\epsilon}, \end{aligned}$$

$$\begin{aligned} M_{10} &\equiv \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \rightarrow \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2+\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2+\epsilon}} \int Dz \left(\frac{g_{\rho\nu}}{[3z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(3z)_\nu(3z)_\rho}{[3z]^{2-\epsilon}} \right) \frac{1}{[2z]^{2-2\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2+\epsilon}} \int Dz \left(\frac{2g_{\rho\nu}}{[3z]^{1-\epsilon}} - \partial_\nu^{(3)} \frac{(3z)_\rho}{[3z]^{1-\epsilon}} \right) \frac{1}{[2z]^{2-2\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \frac{4(12)_\nu}{[12]^{2+\epsilon}} \int Dz \frac{1}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}} - \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2+\epsilon}} \partial_\nu^{(3)} \int Dz \frac{(3z)_\rho}{[3z]^{1-\epsilon}} \frac{1}{[2z]^{2-2\epsilon}} = \end{aligned}$$

$$\begin{aligned}
& \frac{(31)_\nu}{[31]^2} \frac{4(12)_\nu}{[12]^{2+\epsilon}} \frac{A(1-\epsilon, 2-2\epsilon, 1+\epsilon)}{[23]^{1-2\epsilon}} - \frac{1}{2\epsilon} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \partial_\nu^{(3)} \partial_\rho^{(3)} \int Dz \frac{1}{[3z]^{-\epsilon}} \frac{1}{[2z]^{2-2\epsilon}} = \\
& \frac{(31)_\nu}{[31]^2} \frac{4(12)_\nu}{[12]^{2+\epsilon}} \frac{A(1-\epsilon, 2-2\epsilon, 1+\epsilon)}{[23]^{1-2\epsilon}} - \frac{1}{2\epsilon} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2+\epsilon}} \partial_\nu^{(3)} \partial_\rho^{(3)} \frac{A(-\epsilon, 2-2\epsilon, 2+\epsilon)}{[23]^{-2\epsilon}} = \\
& \frac{A(1-\epsilon, 1-2\epsilon, 1+\epsilon)}{\epsilon(1-2\epsilon)} \frac{4(31)_\nu(12)_\nu}{[12]^{2+\epsilon}[23]^{1-2\epsilon}[31]^2} \\
& - \frac{A(1-\epsilon, 1-2\epsilon, 1+\epsilon)}{4\epsilon^2(1-2\epsilon)(1+\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2+\epsilon}} \partial_\nu^{(3)} \partial_\rho^{(3)} \frac{1}{[23]^{-2\epsilon}},
\end{aligned}$$

$$\begin{aligned}
M_{12} \equiv \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{2(1y)_\rho}{[2y]^{2-2\epsilon}[1y]^{2-\epsilon}} \rightarrow \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{2(1y)_\rho}{[2y]^{2-2\epsilon}[1y]^{2+\epsilon}} = \\
- \frac{1}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \partial_\rho^{(1)} \int Dy \frac{1}{[2y]^{2-2\epsilon}[1y]^{1+\epsilon}} = \\
- \frac{1}{1+\epsilon} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \partial_\rho^{(1)} \frac{A(1+\epsilon, 2-2\epsilon, 1-\epsilon)}{[12]} = \\
- \frac{A(1+\epsilon, 1-2\epsilon, 1-\epsilon)}{\epsilon(1+\epsilon)(1-2\epsilon)} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \partial_\rho^{(1)} \frac{1}{[12]}.
\end{aligned}$$

Working with all these expressions in a program written in *Mathematica*, it results that M_1 is not singular and in the limit $\epsilon \rightarrow 0$

$$\begin{aligned}
M_1 \equiv & k \left[\frac{-1/4}{[23]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{5/4}{[12]^2[31]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{-1}{[12]^2[23][31]} + \frac{-1/4}{[12]^2[23]^2} \right] \\
& + k \left[\frac{1/4}{[23]^2[31]^2} + \frac{1/4}{[12][23][31]^2} + \frac{-1/2}{[12]^2[31]^2} + \frac{-1/2}{[12][23]^2[31]} + \frac{1/4}{[12]^2[23][31]} + \frac{1/4}{[12]^2[23]^2} \right] \ln[12] \\
& + k \left[\frac{-1/4}{[23]^2[31]^2} + \frac{-1/4}{[12][23][31]^2} + \frac{1/2}{[12]^2[31]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{-1/4}{[12]^2[23][31]} + \frac{-1/4}{[12]^2[23]^2} \right] \ln[23].
\end{aligned}$$

Now we calculate double finite integrals. All together, the finite integrals from J_1 , J_2 , J_3 , and J_4 can be organized as

$$K_1 \equiv \frac{k}{2} J_{12} + \frac{1}{2} J_2 + \frac{1}{4} J_{31}.$$

The first double integral to calculate is

$$\begin{aligned}
J_{31} \equiv & \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[2y][2z][yz][1y]^2} \rightarrow \\
& \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[1y]^2[2z]^{1-2\epsilon}[2y]^{1-2\epsilon}[yz]} = \\
& \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{-4\epsilon(1-2\epsilon)2(1y)_\rho}{[1y]^2[2z]^{2-2\epsilon}[2y]^{1-2\epsilon}[yz]} \\
& + \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{-4\epsilon(1-2\epsilon)2(1y)_\rho}{[1y]^2[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}[yz]}
\end{aligned}$$

$$\begin{aligned}
& + \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{8(1-2\epsilon)^2 2(1y)_\rho (2z)_\lambda (2y)_\lambda}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{2-2\epsilon} [yz]} = \\
& \quad \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(4-20\epsilon+24\epsilon^2) 2(1y)_\rho}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz]} \\
& \quad + \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(4-20\epsilon+24\epsilon^2) 2(1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz]} \\
& \quad - \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{4(1-2\epsilon)^2 2(1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{2-2\epsilon}} \\
& \equiv (4-20\epsilon+24\epsilon^2) J_{311} + (4-20\epsilon+24\epsilon^2) J_{312} - 4(1-2\epsilon)^2 J_{313} .
\end{aligned}$$

We present a set of integrals by using formulas of Ref. [5],

$$\begin{aligned}
J_{312} & \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz]} \\
& = \frac{(31)_\nu}{[31]^2} \int Dy Dz \left(\frac{g_{\rho\nu}}{[3z]} + \frac{2}{1-2\epsilon} \frac{(3z)_\rho (3z)_\nu}{[3z]^2} \right) \frac{2(1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz]} \\
& = \frac{(31)_\nu}{[31]^2} \int Dy Dz \left(\frac{2-2\epsilon}{1-2\epsilon} \frac{g_{\rho\nu}}{[3z]} - \frac{1}{1-2\epsilon} \partial_\nu^{(3)} \frac{(3z)_\rho}{[3z]} \right) \frac{2(1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz]} \\
& \quad = \frac{2-2\epsilon}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \int Dy Dz \frac{2(1y)_\nu}{[3z][1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz]} \\
& \quad - \frac{1}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy Dz \frac{2(3z)_\rho (1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz][3z]} = \\
& = -\frac{(2-2\epsilon)A(2-2\epsilon, 1, 1)}{1-2\epsilon} \left[\frac{1}{[31][23]^{1-\epsilon}} J(2, 2-3\epsilon, \epsilon) + \frac{1}{[31]^2 [23]^{1-\epsilon}} J(1, 2-3\epsilon, \epsilon) \right. \\
& \quad \left. - \frac{1}{[31]^2 [23]^{1-\epsilon}} J(2, 2-3\epsilon, \epsilon-1) \right] \\
& \quad - \frac{1}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \left[\int Dy Dz \frac{[3z] + [3y] - [yz]}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz][3z]} \right. \\
& \quad \left. + \int Dy Dz \frac{2(3z)_\rho (13)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{1-2\epsilon} [yz][3z]} \right] \\
& = -\frac{(2-2\epsilon)A(2-2\epsilon, 1, 1)}{1-2\epsilon} \left[\frac{1}{[31][23]^{1-\epsilon}} J(2, 2-3\epsilon, \epsilon) + \frac{1}{[31]^2 [23]^{1-\epsilon}} J(1, 2-3\epsilon, \epsilon) \right. \\
& \quad \left. - \frac{1}{[31]^2 [23]^{1-\epsilon}} J(2, 2-3\epsilon, \epsilon-1) \right] \\
& \quad - \frac{A(1, 1, 2-2\epsilon)}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \left[\frac{1}{[23]^{1-\epsilon}} J(2, 2-3\epsilon, \epsilon-1) - \frac{1}{[23]^{1-\epsilon}} J(2, 1-2\epsilon, 0) \right] \\
& \quad + \frac{A(1, 2, 1-2\epsilon)}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \left[\frac{[13]}{[12]^{1-\epsilon}} J(1+\epsilon, 2-3\epsilon, 1) - \frac{1}{[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right], \\
J_{311} & \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz]} =
\end{aligned}$$

$$\begin{aligned}
& \frac{(31)_\nu}{[31]^2} \int Dy Dz \left(\frac{2-2\epsilon}{1-2\epsilon} \frac{g_{\rho\nu}}{[3z]} - \frac{1}{1-2\epsilon} \partial_\nu^{(3)} \frac{(3z)_\rho}{[3z]} \right) \frac{2(1y)_\rho}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz]} = \\
& \quad \frac{2-2\epsilon}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \int Dy Dz \frac{2(1y)_\nu}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz][3z]} \\
& \quad - \frac{1}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy Dz \frac{2(3z)_\rho (1y)_\rho}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz][3z]} = \\
& \quad \frac{2-2\epsilon}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \int Dy Dz \frac{2(1y)_\nu}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz][3z]} \\
& \quad - \frac{1}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy Dz \frac{2(3z)_\rho (3y)_\rho}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz][3z]} \\
& \quad - \frac{1}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy Dz \frac{2(3z)_\rho (13)_\rho}{[1y]^2 [2z]^{1-2\epsilon} [2y]^{2-2\epsilon} [yz][3z]} = \\
& \quad - \frac{2-2\epsilon}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(1)} \frac{A(1, 1, 2-2\epsilon)}{[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \\
& \quad - \frac{A(1, 1, 2-2\epsilon)}{1-2\epsilon} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \left[\left(1 + \epsilon - (31)_\nu \partial_\nu^{(1)} \right) \frac{1}{[12]^{1-\epsilon}} J(\epsilon, 2-3\epsilon, 1) \right. \\
& \quad \left. + \frac{1-\epsilon}{[12]^{2-\epsilon}} J(\epsilon-1, 2-3\epsilon, 1) - \frac{1-\epsilon}{[12]^{2-\epsilon}} J(\epsilon, 1-3\epsilon, 1) \right].
\end{aligned}$$

The following formulas derived by IBP are necessary to obtain the above representation,

$$\begin{aligned}
J(\epsilon-1, 2-3\epsilon, 1) &= (2-3\epsilon) \left[J(\epsilon-1, 3-3\epsilon, 0) - [23] J(\epsilon-1, 3-3\epsilon, 1) \right] \\
&\quad + (\epsilon-1) \left[J(\epsilon, 2-3\epsilon, 0) - [13] J(\epsilon, 2-3\epsilon, 1) \right] \\
J(\epsilon, 1-3\epsilon, 1) &= (1-3\epsilon) \left[J(\epsilon, 2-3\epsilon, 0) - [23] J(\epsilon, 2-3\epsilon, 1) \right] \\
&\quad + \epsilon \left[J(1+\epsilon, 1-3\epsilon, 0) - [13] J(1+\epsilon, 1-3\epsilon, 1) \right]. \tag{6}
\end{aligned}$$

The third integral in J_{31} is simple,

$$\begin{aligned}
J_{313} &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(1y)_\rho}{[1y]^2 [2z]^{2-2\epsilon} [2y]^{2-2\epsilon}} = \\
&\quad - \frac{(31)_\nu}{[31]^2} \partial_\rho^{(1)} \frac{A(1, 2-2\epsilon, 1)}{[12]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} = \\
&\quad - \frac{(31)_\nu}{[31]^2} \partial_\rho^{(1)} \frac{A(1, 2-2\epsilon, 1)}{[12]^{1-\epsilon}} \left[\frac{2-2\epsilon}{1-2\epsilon} g_{\rho\nu} J(0, 2-2\epsilon, 1) + \frac{1}{1-2\epsilon} \partial_\nu^{(3)} (23)_\rho J(0, 2-2\epsilon, 1) \right. \\
&\quad \left. + \frac{1}{2(1-2\epsilon)^2} \partial_\nu^{(3)} \partial_\rho^{(2)} J(0, 1-2\epsilon, 1) \right].
\end{aligned}$$

All these formulas can be programmed in *Mathematica* and the result for J_{31} is

$$J_{31} = \left[\frac{-8}{[12][23]^2} + \frac{-4}{[12]^2[31]} + \frac{4}{[23]^2[31]} + \frac{4[31]}{[12]^2[23]^2} \right] J[1, 1, 1]$$

$$\begin{aligned}
& + \left[\frac{-4}{[12]^2[23]^2} + \frac{4}{[12][23]^2[31]} + \frac{4}{[12]^2[23][31]} \right] \ln[12] + \frac{-8}{[12]^2[23][31]} \ln[23] \\
& + \left[\frac{4}{[12]^2[23]^2} + \frac{-4}{[12][23]^2[31]} + \frac{4}{[12]^2[23][31]} \right] \ln[31] .
\end{aligned}$$

The second finite double integral to calculate is

$$2J_{21} \equiv 2 \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \partial_\rho^{(1)} \partial_\rho^{(3)} \int Dy Dz \frac{(2y)_\sigma (2z)_\sigma}{[2y]^{2-2\epsilon} [2z]^{1-2\epsilon} [yz][1y][3z]} .$$

This is seen as a differential operator applied to the integral

$$\begin{aligned}
& 2 \int Dy Dz \frac{(2y)_\sigma (2z)_\sigma}{[2y]^{2-2\epsilon} [2z]^{1-2\epsilon} [yz][1y][3z]} = \\
& \int Dy Dz \frac{1}{[2y]^{1-2\epsilon} [2z]^{1-2\epsilon} [yz][1y][3z]} + \int Dy Dz \frac{1}{[2y]^{2-2\epsilon} [2z]^{-2\epsilon} [yz][1y][3z]} \\
& - \int Dy Dz \frac{1}{[2y]^{2-2\epsilon} [2z]^{1-2\epsilon} [1y][3z]} \equiv I_1 + I_2 - I_3 .
\end{aligned}$$

I_1 is finite, and the difference $I_2 - I_3$ must give a finite result:

$$I_2 - I_3 = \frac{A(1, 1, 1 - 2\epsilon)}{\epsilon(1 - 2\epsilon)} \frac{1}{[12]^{1-\epsilon}} J(\epsilon, 1 - 3\epsilon, 1) + \frac{A^2(1, 1, 1 - 2\epsilon)}{\epsilon^2(1 - 2\epsilon)} \frac{1}{[12]^{1-\epsilon} [23]^{-\epsilon}} .$$

Taking into account this relation, Eq. (6), and the relation

$$\begin{aligned}
& \partial_\rho^{(1)} \partial_\rho^{(3)} I_1 = \int Dy Dz \frac{4(1y)_\rho (3z)_\rho}{[2y]^{1-2\epsilon} [2z]^{1-2\epsilon} [yz][1y]^2 [3z]^2} = \\
& 2A(1, 1 - 2\epsilon, 2) \left[-\frac{[31]}{[12]^{1-\epsilon}} J(1 + \epsilon, 1 - 3\epsilon, 2) + \frac{1}{[12]^{1-\epsilon}} J(\epsilon, 1 - 3\epsilon, 2) \right. \\
& \left. + \frac{1}{[23]^{1-\epsilon}} J(2, 1 - 3\epsilon, \epsilon) - \frac{1}{[12]^{1-\epsilon}} J(0, 1 - 3\epsilon, 2) \right] \equiv I_4 ,
\end{aligned}$$

we obtain by *Mathematica*

$$\begin{aligned}
2J_{21} &= \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \left[\partial_\rho^{(1)} \partial_\rho^{(3)} (I_2 - I_3) + I_4 \right] = \\
& \left[\frac{-2}{[12]^2[23]^2} + \frac{-2}{[12]^2[31]^2} + \frac{-2}{[23]^2[31]^2} + \frac{-4}{[12][23][31]^2} + \frac{4}{[12][23]^2[31]} + \frac{4}{[12]^2[23][31]} \right] \\
& + \left[\frac{2}{[12][23]^2} + \frac{-2}{[12][31]^2} + \frac{2[12]}{[23]^2[31]^2} + \frac{-4}{[23][31]^2} + \frac{-4}{[23]^2[31]} \right] J(1, 1, 1) \\
& + \left[\frac{2}{[23]^2[31]^2} + \frac{2}{[12][23][31]^2} + \frac{-2}{[12][23]^2[31]} \right] \ln[12] \\
& + \left[\frac{2}{[12]^2[23]^2} + \frac{2}{[23]^2[31]^2} + \frac{-2}{[12][23][31]^2} + \frac{-4}{[12][23]^2[31]} + \frac{-2}{[12]^2[23][31]} \right] \ln[23] \\
& + \left[\frac{-2}{[12]^2[23]^2} + \frac{-4}{[23]^2[31]^2} + \frac{6}{[12][23]^2[31]} + \frac{2}{[12]^2[23][31]} \right] \ln[31] .
\end{aligned}$$

The first finite single integral is

$$\begin{aligned}
J_{12} = & \frac{(31)_\nu}{[31]^2} \partial_\rho^{(1)} \frac{1}{[21]} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[1z][2z]} = \\
& \left[\frac{2}{[12][31]^2} + \frac{-2[23]}{[12]^2[31]^2} + \frac{2}{[12]^2[31]} \right] J(1, 1, 1) \\
& + \left[\frac{1}{[12]^2[31]^2} + \frac{1}{[12][23][31]^2} + \frac{-1}{[12]^2[23][31]} \right] \ln [12] + \frac{-2}{[12]^2[31]^2} \ln [23] \\
& + \left[\frac{1}{[12]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{1}{[12]^2[23][31]} \right] \ln [31] .
\end{aligned}$$

The second single finite integral is

$$\begin{aligned}
2J_{22} = & 2 \frac{(31)_\nu}{[31]^2} \frac{(23)_\sigma}{[23]^{1-\epsilon}} \partial_\nu^{(1)} \int Dy \frac{(2y)_\sigma}{[2y]^2[y3][1y]} = \frac{2}{[12][23][31]^2} + \frac{1}{[23][31]^2} J(1, 1, 1) \\
& + \left[\frac{-1}{[12]^2[31]^2} + \frac{1}{[12]^2[23][31]} \right] \ln [23] + \left[\frac{1}{[12]^2[31]^2} + \frac{-1}{[12]^2[23][31]} \right] \ln [31] .
\end{aligned}$$

Taking into account the contribution of singular integrals, M_1 , we obtain

$$\begin{aligned}
& -T_{111} = K_1 + M_1 = \\
& k \left[\frac{-1/4}{[12]^2[23]^2} + \frac{5/4}{[12]^2[31]^2} + \frac{-1/4}{[23]^2[31]^2} + \frac{-2}{[12][23][31]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{-1}{[12]^2[23][31]} \right] \\
& + k \left[\frac{1}{[12][31]^2} + \frac{-1/2}{[23][31]^2} + \frac{-[23]}{[12]^2[31]^2} + \frac{1}{[12]^2[31]} \right] J(1, 1, 1) \\
& + k \left[\frac{1/4}{[12]^2[23]^2} + \frac{1/4}{[23]^2[31]^2} + \frac{3/4}{[12][23][31]^2} + \frac{-1/2}{[12][23]^2[31]} + \frac{-1/4}{[12]^2[23][31]} \right] \ln [12] \\
& + k \left[\frac{-1/4}{[12]^2[23]^2} + \frac{-1/4}{[23]^2[31]^2} + \frac{-1/4}{[12][23][31]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{-3/4}{[12]^2[23][31]} \right] \ln [23] \\
& + k \left[\frac{-1/2}{[12][23][31]^2} + \frac{1}{[12]^2[23][31]} \right] \ln [31] \\
& + \left[\frac{-1}{[12]^2[23]^2} + \frac{-1}{[12]^2[31]^2} + \frac{-1}{[23]^2[31]^2} + \frac{-2}{[12][23][31]^2} + \frac{2}{[12][23]^2[31]} + \frac{2}{[12]^2[23][31]} \right] \\
& + \left[\frac{-1}{[12][23]^2} + \frac{-1}{[12][31]^2} + \frac{[12]}{[23]^2[31]^2} + \frac{-2}{[23][31]^2} \right. \\
& \quad \left. + \frac{-1}{[12]^2[31]} + \frac{-1}{[23]^2[31]} + \frac{[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\
& + \left[\frac{-1}{[12]^2[23]^2} + \frac{1}{[23]^2[31]^2} + \frac{1}{[12][23][31]^2} + \frac{1}{[12]^2[23][31]} \right] \ln [12] \\
& + \left[\frac{1}{[12]^2[23]^2} + \frac{1}{[23]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{-2}{[12][23]^2[31]} + \frac{-3}{[12]^2[23][31]} \right] \ln [23] \\
& + \left[\frac{-2}{[23]^2[31]^2} + \frac{2}{[12][23]^2[31]} + \frac{2}{[12]^2[23][31]} \right] \ln [31] . \tag{7}
\end{aligned}$$

3.1.2 Calculation of T_{112}

$$\begin{aligned}
T_{112} = & -\frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
& k \left[\frac{-1/16}{[12]^2 [31]^2} + \frac{1/16}{[12]^2 [31]^2} + \frac{3/16}{[23]^2 [31]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{-1/8}{[12][23]^2 [31]} \right] \\
& + k \left[\frac{1/4}{[12][31]^2} + \frac{-1/8[12]}{[23]^2 [31]^2} + \frac{1/8}{[23]^2 [31]} \right] J(1, 1, 1) \\
& + k \left[\frac{1/4}{[23]^2 [31]^2} + \frac{3/8}{[12][23][31]^2} + \frac{1/8}{[12][23]^2 [31]} \right] \ln[12] \\
& + k \left[\frac{-1/16}{[12]^2 [23]^2} + \frac{-3/8}{[12]^2 [31]^2} + \frac{-5/16}{[23]^2 [31]^2} + \frac{-1/16}{[12][23][31]^2} \right. \\
& \quad \left. + \frac{3/8}{[12][23]^2 [31]} + \frac{-1/16}{[12]^2 [23][31]} \right] \ln[23] \\
& + k \left[\frac{1/16}{[12]^2 [23]^2} + \frac{3/8}{[12]^2 [31]^2} + \frac{1/16}{[23]^2 [31]^2} + \frac{-5/16}{[12][23][31]^2} \right. \\
& \quad \left. + \frac{-1/2}{[12][23]^2 [31]} + \frac{1/16}{[12]^2 [23][31]} \right] \ln[31] \\
& + \left[\frac{1/2}{[12]^2 [23]^2} + \frac{-1/2}{[12][23]^2 [31]} + \frac{-1/2}{[12]^2 [23][31]} \right] + \left[\frac{-1}{[12][23]^2} + \frac{1}{[23]^2 [31]} \right] J(1, 1, 1) \\
& + \left[\frac{-1/2}{[12]^2 [23]^2} + \frac{1/2}{[12][23]^2 [31]} + \frac{-1/2}{[12]^2 [23][31]} \right] \ln[23] \\
& + \left[\frac{1/2}{[12]^2 [23]^2} + \frac{-1/2}{[12][23]^2 [31]} + \frac{1/2}{[12]^2 [23][31]} \right] \ln[31] . \tag{8}
\end{aligned}$$

Details of this calculation are explained in Appendix A.

3.1.3 Calculation of T_{12} and T_{13}

Eq. (4) for T_1 contains two finite single integrals, T_{12} and T_{13} :

$$\begin{aligned}
T_{13} \equiv & \frac{k}{4} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^2} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(1z)_\rho}{[1z]^{1-\epsilon}} = \\
& k \left[\frac{1/4}{[12]^2 [23]^2} + \frac{1/4}{[12]^2 [31]^2} + \frac{1/4}{[23]^2 [31]^2} + \frac{-1/2}{[12][23]^2 [31]} \right] \\
& + k \left[\frac{1/4}{[12][31]^2} + \frac{-1/2[23]}{[12]^2 [31]^2} + \frac{1/4}{[12]^2 [31]} \right] J(1, 1, 1) \\
& + k \left[\frac{1/8}{[12]^2 [23]^2} + \frac{-1/2}{[12]^2 [31]^2} + \frac{-1/8}{[23]^2 [31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{1/8}{[12]^2 [23][31]} \right] \ln[12] \\
& + k \left[\frac{1}{[12]^2 [31]^2} \right] \ln[23]
\end{aligned}$$

$$+k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{-1/2}{[12]^2[31]^2} + \frac{1/8}{[23]^2[31]^2} + \frac{1/8}{[12][23][31]^2} + \frac{-1/8}{[12]^2[23][31]} \right] \ln [31] . \quad (9)$$

The second finite integral in Eq. (4) is

$$T_{12} \equiv \frac{k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} = \\ k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{1/4}{[12]^2[31]^2} + \frac{-1/8}{[23]^2[31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{1/4}{[12][23]^2[31]} + \frac{-1/8}{[12]^2[23][31]} \right] . \quad (10)$$

3.1.4 Total result for T_1

Combining Eqs. (7), (8), (9) and (10), we obtain for T_1

$$T_1 = T_{111} + T_{112} + T_{12} + T_{13} = \\ k \left[\frac{5/16}{[12]^2[23]^2} + \frac{-11/16}{[12]^2[31]^2} + \frac{9/16}{[23]^2[31]^2} + \frac{11/8}{[12][23][31]^2} + \frac{-7/8}{[12][23]^2[31]} + \frac{7/8}{[12]^2[23][31]} \right] \\ + k \left[\frac{-1/2}{[12][31]^2} + \frac{-1/8[12]}{[23]^2[31]^2} + \frac{1/2}{[23][31]^2} + \frac{1/2[23]}{[12]^2[31]^2} + \frac{-3/4}{[12]^2[31]} + \frac{1/8}{[23]^2[31]} \right] J(1, 1, 1) \\ + k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{-1/2}{[12]^2[23]^2} + \frac{-1/8}{[23]^2[31]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{5/8}{[12][23]^2[31]} + \frac{3/8}{[12]^2[23][31]} \right] \ln [12] \\ + k \left[\frac{3/16}{[12]^2[23]^2} + \frac{5/8}{[12]^2[31]^2} + \frac{-1/16}{[23]^2[31]^2} + \frac{3/16}{[12][23][31]^2} + \frac{-1/8}{[12][23]^2[31]} + \frac{11/16}{[12]^2[23][31]} \right] \ln [23] \\ + k \left[\frac{-1/16}{[12]^2[23]^2} + \frac{-1/8}{[12]^2[31]^2} + \frac{3/16}{[23]^2[31]^2} + \frac{5/16}{[12][23][31]^2} + \frac{-1/2}{[12][23]^2[31]} + \frac{-17/16}{[12]^2[23][31]} \right] \ln [31] \\ + \left[\frac{3/2}{[12]^2[23]^2} + \frac{1}{[12]^2[31]^2} + \frac{1}{[23]^2[31]^2} + \frac{2}{[12][23][31]^2} + \frac{-5/2}{[12][23]^2[31]} + \frac{-5/2}{[12]^2[23][31]} \right] \\ + \left[\frac{1}{[12][31]^2} + \frac{-[12]}{[23]^2[31]^2} + \frac{2}{[23][31]^2} + \frac{1}{[12]^2[31]} + \frac{2}{[23]^2[31]} + \frac{-[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\ + \left[\frac{1}{[12]^2[23]^2} + \frac{-1}{[23]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{-1}{[12]^2[23][31]} \right] \ln [12] \\ + \left[\frac{-3/2}{[12]^2[23]^2} + \frac{-1}{[23]^2[31]^2} + \frac{1}{[12][23][31]^2} + \frac{5/2}{[12][23]^2[31]} + \frac{5/2}{[12]^2[23][31]} \right] \ln [23] \\ + \left[\frac{1/2}{[12]^2[23]^2} + \frac{2}{[23]^2[31]^2} + \frac{-5/2}{[12][23]^2[31]} + \frac{-3/2}{[12]^2[23][31]} \right] \ln [31] .$$

As one can see from Eq. (1), T_1 contributes to the full result T of diagram (c) with factor (-2).

3.2 The second line of Eq. (1)

$$T_2 \equiv \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\sigma}(z2) \Pi_{\mu\lambda}(zy) . \quad (11)$$

As one can see from Eq. (1), integral T_2 contributes to the total result for diagram c with factor 2. We use simple algebra to represent T_2 as combination of terms with simpler Lorentz structure.

$$\Pi_{\mu\lambda}(zy) \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} = \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{1}{4(1-\epsilon)} \left[\partial_{(y)}^2 \left(\frac{(yz)_\mu}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) - \left(\partial_{(y)}^2 \frac{(yz)_\mu}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} - \frac{(yz)_\mu}{[yz]^{1-\epsilon}} \left(\partial_{(y)}^2 \frac{1}{[1y]^{1-\epsilon}} \right) \right]. \quad (12)$$

Eq. (11) can be transformed to a form

$$\begin{aligned} -T_2 &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\lambda}(zy) = \\ &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \right. \\ &- \frac{1}{4(1-\epsilon)} \left[\partial_{(y)}^2 \left(\frac{(yz)_\mu}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) - \left(\partial_{(y)}^2 \frac{(yz)_\mu}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} - \frac{(yz)_\mu}{[yz]^{1-\epsilon}} \left(\partial_{(y)}^2 \frac{1}{[1y]^{1-\epsilon}} \right) \right] \Big\} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \times \\ &\times \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} + \frac{1}{4(1-\epsilon)} \left(\partial_{(y)}^2 \frac{(yz)_\mu}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \right\} - \\ &- \frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \partial_{(y)}^2 \frac{(yz)_\mu}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} + \\ &+ \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(yz)_\mu}{[yz]^{1-\epsilon}} \delta^{(4-2\epsilon)}(1y) = \\ &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\ &+ \frac{1}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \int Dy \frac{1}{[2y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \partial_{(y)}^2 \frac{(yz)_\mu}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} + \\ &+ \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]^{1-\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\ &+ \frac{1}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \int Dy k \delta(2y) \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(yz)_\mu}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} + \\ &+ \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]^{1-\epsilon}} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\ &+ \frac{k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(2z)_\mu}{[2z]^{1-\epsilon}} + \\ &+ \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]^{1-\epsilon}} = \end{aligned}$$

$$\begin{aligned}
& \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\
& - \frac{k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\sigma}(z2) \frac{(2z)_\mu}{[2z]^{1-\epsilon}} + \\
& + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \Pi_{\rho\sigma}(z2) \frac{1}{[2z]^{1-\epsilon}} + \\
& + \frac{k}{4} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]} = \\
& \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\
& - \frac{k}{8} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\sigma}(z2) \frac{(2z)_\mu}{[2z]} + \\
& + \frac{3k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\rho^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} + \\
& + \frac{k}{4} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^2} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]} = \\
& \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\
& - \frac{k}{8} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\sigma}(z2) \frac{(2z)_\mu}{[2z]} + \\
& + \frac{3k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \\
& + \frac{k}{4} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^2} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]} \equiv \\
& \equiv T_{21} + T_{22} + T_{23} + T_{24} . \quad (13)
\end{aligned}$$

Integrals T_{22} and T_{24} are finite, integral T_{23} is singular. Since integral T_2 is finite, $T_{21} + T_{23}$ should be finite. The sum $T_{21} + T_{23}$ is calculated in Appendix B.

3.2.1 Calculation of T_{22} and T_{24}

In Eq. (13) for T_2 there are two finite single integrals,

$$\begin{aligned}
T_{24} = & \frac{k}{4} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^2} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \frac{(1z)_\mu}{[1z]} = \\
& k \left[\frac{1/4}{[12]^2[31]^2} + \frac{1/4}{[12][23][31]^2} + \frac{-1/4}{[12]^2[23][31]} \right] \\
& + k \left[\frac{1/4}{[12][31]^2} + \frac{-1/2[23]}{[12]^2[31]^2} + \frac{1/2}{[12]^2[31]} \right] J(1, 1, 1) \\
& + k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{1/2}{[12]^2[31]^2} + \frac{-1/8}{[23]^2[31]^2} + \frac{-1/8}{[12][23][31]^2} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1/4}{[12][23]^2[31]} + \frac{-3/8}{[12]^2[23][31]} \Big] \ln[12] \\
& + k \left[\frac{1/8}{[12]^2[23]^2} + \frac{-1/2}{[12]^2[31]^2} + \frac{1/8}{[23]^2[31]^2} + \frac{1/8}{[12][23][31]^2} \right. \\
& \quad \left. + \frac{-1/4}{[12][23]^2[31]} + \frac{3/8}{[12]^2[23][31]} \right] \ln[31], \tag{14}
\end{aligned}$$

and

$$\begin{aligned}
T_{22} = & -\frac{k}{8} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\rho\sigma}(z2) \frac{(2z)_\mu}{[2z]} = \tag{15} \\
& k \left[\frac{3/8}{[12]^2[23]^2} + \frac{-3/4}{[12]^2[31]^2} + \frac{3/8}{[12]^2[31]^2} + \frac{3/8}{[12][23][31]^2} + \frac{-3/4}{[12][23]^2[31]} + \frac{3/8}{[12]^2[23][31]} \right].
\end{aligned}$$

3.2.2 Total result for T_2

Combining T_{22}, T_{24} with the result of Appendix B for $T_{21} + T_{23}$,

$$\begin{aligned}
& -T_2 = T_{22} + T_{24} + K_3 + M_3 = \\
& k \left[\frac{11/16}{[12]^2[23]^2} + \frac{-9/16}{[12]^2[31]^2} + \frac{7/16}{[23]^2[31]^2} + \frac{5/8}{[12][23][31]^2} + \frac{-9/8}{[12][23]^2[31]} + \frac{-1/8}{[12]^2[23][31]} \right] \\
& + k \left[\frac{1/4}{[12][31]^2} + \frac{1/4[12]}{[23]^2[31]^2} + \frac{-1/4}{[23][31]^2} + \frac{-1/2[23]}{[12]^2[31]^2} + \frac{1/2}{[12]^2[31]} + \frac{-1/4}{[23]^2[31]} \right] J(1, 1, 1) \\
& + k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{1/2}{[12]^2[31]^2} + \frac{1/8}{[23]^2[31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{1/4}{[12][23]^2[31]} + \frac{-3/8}{[12]^2[23][31]} \right] \ln[12] \\
& + k \left[\frac{-3/16}{[12]^2[23]^2} + \frac{3/8}{[12]^2[31]^2} + \frac{1/16}{[23]^2[31]^2} + \frac{-11/16}{[12][23][31]^2} + \frac{1/8}{[12][23]^2[31]} + \frac{-3/16}{[12]^2[23][31]} \right] \ln[23] \\
& + k \left[\frac{5/16}{[12]^2[23]^2} + \frac{-7/8}{[12]^2[31]^2} + \frac{-3/16}{[23]^2[31]^2} + \frac{13/16}{[12][23][31]^2} + \frac{-3/8}{[12][23]^2[31]} + \frac{9/16}{[12]^2[23][31]} \right] \ln[31] \\
& + \left[\frac{1/2}{[12]^2[23]^2} + \frac{-1}{[23]^2[31]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{3/2}{[12]^2[23][31]} \right] \\
& + \left[\frac{-3}{[12][23]^2} + \frac{1}{[12][31]^2} + \frac{1/2[12]}{[23]^2[31]^2} + \frac{-1}{[23][31]^2} + \frac{-2}{[12]^2[31]} + \frac{3/2}{[23]^2[31]} + \frac{[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\
& + \left[\frac{-2}{[12]^2[23]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{5/2}{[12][23]^2[31]} + \frac{1}{[12]^2[23][31]} \right] \ln[12] \\
& + \left[\frac{-1/2}{[12]^2[23]^2} + \frac{1}{[23]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{-1/2}{[12][23]^2[31]} + \frac{-5/2}{[12]^2[23][31]} \right] \ln[23] \\
& + \left[\frac{5/2}{[12]^2[23]^2} + \frac{-1}{[23]^2[31]^2} + \frac{3/2}{[12][23][31]^2} + \frac{-2}{[12][23]^2[31]} + \frac{3/2}{[12]^2[23][31]} \right] \ln[31].
\end{aligned}$$

3.3 The third line of (1)

$$T_3 \equiv \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{(1y)_\lambda}{[1y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\rho\lambda}(zy) \Pi_{\mu\nu}(z3). \tag{16}$$

Integral T_3 contributes to the total result for diagram (c) with factor 2. We use simple algebra to represent integral (16) as combination of terms with simpler Lorentz structure, by using for $\Pi_{\rho\lambda}(zy)(1y)_\lambda/[1y]^{2-\epsilon}$ the form of Eq. (3). It can be checked that expression (16) can be transformed identically as in the case of T_1 , Eqs. (2)-(4), with the only substitutions $\Pi_{\rho\nu}(z3) \mapsto \Pi_{\rho\sigma}(z2)$ and $\Pi_{\mu\sigma}(z2) \mapsto \Pi_{\mu\nu}(z3)$. This leads to a form completely analogous to Eq. (4)

$$\begin{aligned}
T_3 = & \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \left\{ \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\rho}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} + \\
& + \frac{k}{8(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]^{1-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} + \\
& + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^{2-\epsilon}} \frac{(21)_\sigma}{[12]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \frac{(1z)_\rho}{[1z]^{1-\epsilon}} \equiv \\
& \equiv T_{31} + T_{32} + T_{33} \equiv T_{311} + T_{312} + T_{32} + T_{33} .(17)
\end{aligned}$$

Single integrals T_{32} and T_{33} are finite. Since T_3 is finite, it means that T_{31} is also finite. Moreover, the finiteness (UV and IR) of T_{31} can be checked by index counting directly. Now we consider T_{31} , $T_{31} \equiv T_{311} + T_{312}$. Both T_{311} and T_{312} are finite. Integrals T_{311} and T_{312} are calculated in Appendices C and D, respectively.

3.3.1 Calculation of T_{32} and T_{33}

Eq. (17) contains two finite integrals,

$$\begin{aligned}
T_{33} \equiv & \frac{k}{4} \frac{(31)_\nu}{[31]^2} \frac{(21)_\sigma}{[12]^2} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \frac{(1z)_\rho}{[1z]} = \\
& k \left[\frac{1/4}{[12]^2[23]^2} + \frac{1/4}{[12]^2[31]^2} + \frac{1/4}{[23]^2[31]^2} + \frac{-1/2}{[12][23]^2[31]} \right] \\
& + k \left[\frac{1/4}{[12][31]^2} + \frac{-1/2[23]}{[12]^2[31]^2} + \frac{1/4}{[12]^2[31]} \right] J(1, 1, 1) \\
& + k \left[\frac{1/8}{[12]^2[23]^2} + \frac{-1/2}{[12]^2[31]^2} + \frac{-1/8}{[23]^2[31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{1/8}{[12]^2[23][31]} \right] \ln[12] \\
& + k \left[\frac{1}{[12]^2[31]^2} \right] \ln[23] \\
& + k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{-1/2}{[12]^2[31]^2} + \frac{1/8}{[23]^2[31]^2} + \frac{1/8}{[12][23][31]^2} + \frac{-1/8}{[12]^2[23][31]} \right] \ln[31], \quad (18)
\end{aligned}$$

and

$$\begin{aligned}
T_{32} \equiv & \frac{k}{8} \frac{(31)_\nu}{[31]^2} \partial_\sigma^{(2)} \frac{1}{[12]} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \frac{(2z)_\rho}{[2z]} = \\
& k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{1/4}{[12]^2[31]^2} + \frac{-1/8}{[23]^2[31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{1/4}{[12][23]^2[31]} + \frac{-1/8}{[12]^2[23][31]} \right] . \quad (19)
\end{aligned}$$

3.3.2 Total result for T_3

Using the result of Appendices C and D, and of Eqs. (18) and (19),

$$\begin{aligned}
T_3 \equiv T_{32} + T_{33} + K_4 + M_4 + K_5 + M_5 = \\
k \left[\frac{5/16}{[12]^2[23]^2} + \frac{9/16}{[12]^2[31]^2} + \frac{1/16}{[23]^2[31]^2} + \frac{3/8}{[12][23][31]^2} + \frac{-3/8}{[12][23]^2[31]} + \frac{-3/8}{[12]^2[23][31]} \right] \\
+ k \left[\frac{1/4[12]}{[23]^2[31]^2} + \frac{-1/4}{[23][31]^2} + \frac{-1/2[23]}{[12]^2[31]^2} + \frac{1/4}{[12]^2[31]} + \frac{-1/4}{[23]^2[31]} \right] J(1, 1, 1) \\
+ k \left[\frac{1/4}{[12]^2[23]^2} + \frac{-3/4}{[12]^2[31]^2} + \frac{-1/4}{[23]^2[31]^2} + \frac{-1/4}{[12][23][31]^2} + \frac{-1/4}{[12][23]^2[31]} + \frac{1/4}{[12]^2[23][31]} \right] \ln[12] \\
+ k \left[\frac{-1/16}{[12]^2[23]^2} + \frac{13/8}{[12]^2[31]^2} + \frac{3/16}{[23]^2[31]^2} + \frac{-1/16}{[12][23][31]^2} + \frac{-1/8}{[12][23]^2[31]} + \frac{-1/16}{[12]^2[23][31]} \right] \ln[23] \\
+ k \left[\frac{-3/16}{[12]^2[23]^2} + \frac{-7/8}{[12]^2[31]^2} + \frac{1/16}{[23]^2[31]^2} + \frac{5/16}{[12][23][31]^2} + \frac{3/8}{[12][23]^2[31]} + \frac{-3/16}{[12]^2[23][31]} \right] \ln[31] \\
+ \left[\frac{5/2}{[12]^2[23]^2} + \frac{1}{[12][23][31]^2} + \frac{-5/2}{[12][23]^2[31]} + \frac{-1/2}{[12]^2[23][31]} \right] \\
+ \left[\frac{1/2[12]}{[23]^2[31]^2} + \frac{-1}{[23][31]^2} + \frac{1/2}{[23]^2[31]} + \frac{-[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\
+ \left[\frac{1/2}{[12][23][31]^2} + \frac{-5/2}{[12][23]^2[31]} + \frac{-1}{[12]^2[23][31]} \right] \ln[12] \\
+ \left[\frac{-5/2}{[12]^2[23]^2} + \frac{5/2}{[12][23]^2[31]} + \frac{-1/2}{[12]^2[23][31]} \right] \ln[23] \\
+ \left[\frac{5/2}{[12]^2[23]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{3/2}{[12]^2[23][31]} \right] \ln[31] .
\end{aligned}$$

4 The result for diagram c

Summing up the contribution for all three lines of Eq. (1), we obtain the result for diagram (c).

$$\begin{aligned}
T \equiv -2T_1 + 2T_2 + 2T_3 = \\
k \left[\frac{-11/8}{[12]^2[23]^2} + \frac{29/8}{[12]^2[31]^2} + \frac{-15/8}{[23]^2[31]^2} + \frac{-13/4}{[12][23][31]^2} + \frac{13/4}{[12][23]^2[31]} + \frac{-9/4}{[12]^2[23][31]} \right] \\
+ k \left[\frac{1/2}{[12][31]^2} + \frac{1/4[12]}{[23]^2[31]^2} + \frac{-1}{[23][31]^2} + \frac{-[23]}{[12]^2[31]^2} + \frac{1}{[12]^2[31]} + \frac{-1/4}{[23]^2[31]} \right] J(1, 1, 1) \\
+ k \left[\frac{1}{[12]^2[23]^2} + \frac{-3/2}{[12]^2[31]^2} + \frac{-1/2}{[23]^2[31]^2} + \frac{3/4}{[12][23][31]^2} + \frac{-9/4}{[12][23]^2[31]} + \frac{1/2}{[12]^2[23][31]} \right] \ln[12] \\
+ k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{5/4}{[12]^2[31]^2} + \frac{3/8}{[23]^2[31]^2} + \frac{7/8}{[12][23][31]^2} + \frac{-1/4}{[12][23]^2[31]} + \frac{-9/8}{[12]^2[23][31]} \right] \ln[23] \\
+ k \left[\frac{-7/8}{[12]^2[23]^2} + \frac{1/4}{[12]^2[31]^2} + \frac{1/8}{[23]^2[31]^2} + \frac{-13/8}{[12][23][31]^2} + \frac{5/2}{[12][23]^2[31]} + \frac{5/8}{[12]^2[23][31]} \right] \ln[31]
\end{aligned}$$

$$\begin{aligned}
& + \left[\frac{1}{[12]^2[23]^2} + \frac{-2}{[12]^2[31]^2} + \frac{-2}{[12][23][31]^2} + \frac{-1}{[12][23]^2[31]} + \frac{1}{[12]^2[23][31]} \right] \\
& + \left[\frac{6}{[12][23]^2} + \frac{-4}{[12][31]^2} + \frac{2[12]}{[23]^2[31]^2} + \frac{-4}{[23][31]^2} + \frac{2}{[12]^2[31]} + \frac{-6}{[23]^2[31]} + \frac{-2[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\
& + \left[\frac{2}{[12]^2[23]^2} + \frac{2}{[23]^2[31]^2} + \frac{4}{[12][23][31]^2} + \frac{-10}{[12][23]^2[31]} + \frac{-2}{[12]^2[23][31]} \right] \ln[12] \\
& + \left[\frac{-1}{[12]^2[23]^2} + \frac{1}{[12][23]^2[31]} + \frac{-1}{[12]^2[23][31]} \right] \ln[23] \\
& + \left[\frac{-1}{[12]^2[23]^2} + \frac{-2}{[23]^2[31]^2} + \frac{-4}{[12][23][31]^2} + \frac{9}{[12][23]^2[31]} + \frac{3}{[12]^2[23][31]} \right] \ln[31] .
\end{aligned}$$

Self-consistency requires the value $k = -4$. This follows from formulas of Ref. [4]. Thus, the result for diagram (c) is

$$\begin{aligned}
T = & \left[\frac{13/2}{[12]^2[23]^2} + \frac{-33/2}{[12]^2[31]^2} + \frac{15/2}{[23]^2[31]^2} + \frac{11}{[12][23][31]^2} + \frac{-14}{[12][23]^2[31]} + \frac{10}{[12]^2[23][31]} \right] \\
& + \left[\frac{6}{[12][23]^2} + \frac{-6}{[12][31]^2} + \frac{[12]}{[23]^2[31]^2} + \frac{4[23]}{[12]^2[31]^2} + \frac{-2}{[12]^2[31]} + \frac{-5}{[23]^2[31]} + \frac{-2[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\
& + \left[\frac{-2}{[12]^2[23]^2} + \frac{6}{[12]^2[31]^2} + \frac{4}{[23]^2[31]^2} + \frac{1}{[12][23][31]^2} + \frac{-1}{[12][23]^2[31]} + \frac{-4}{[12]^2[23][31]} \right] \ln[12] \\
& + \left[\frac{-1/2}{[12]^2[23]^2} + \frac{-5}{[12]^2[31]^2} + \frac{-3/2}{[23]^2[31]^2} + \frac{-7/2}{[12][23][31]^2} + \frac{2}{[12][23]^2[31]} + \frac{7/2}{[12]^2[23][31]} \right] \ln[23] \\
& + \left[\frac{5/2}{[12]^2[23]^2} + \frac{-1}{[12]^2[31]^2} + \frac{-5/2}{[23]^2[31]^2} + \frac{5/2}{[12][23][31]^2} + \frac{-1}{[12][23]^2[31]} + \frac{1/2}{[12]^2[23][31]} \right] \ln[31] .
\end{aligned}$$

5 Conclusion

We have treated in this paper the vertex of gluon self-interaction in the position space, significantly reducing the number of indices of the Lorentz group. This reduction allowed us to obtain the result for the third contribution to the Lcc diagram by using *Mathematica* software. It is clear from the index counting arguments that diagram (c) does not diverge in the limit $\epsilon \rightarrow 0$, due to transversality of the gauge propagator in the Landau gauge. The transversality makes the entire diagram convergent superficially, and makes also its subgraphs convergent. $\mathcal{N} = 4$ supersymmetry does not play any role in the scale independence of the diagrams (a), (b) and (c). This is a pure effect of the Landau gauge, and this result will be true in any gauge theory, for example in pure QCD. $\mathcal{N} = 4$ supersymmetry is important only for cancellation of poles between diagrams (d) and (e)². Scale independence of the kernels of dressed mean fields in the effective action of gauge theories and gravity, and conformal invariance of the effective action of dressed mean fields for these theories can play an important role for different types of theories such as the $\mathcal{N} = 8$ supergravity [50, 51, 52, 53], Chern-Simons theory near the RG fixed points [54], massless gauge theory near fixed points in the coupling space, topological field theories in higher dimensions, finite $\mathcal{N} = 1$ supersymmetric theories [55, 56, 57, 58, 59]. To introduce

²The full result for the sum of all the five two-loop planar diagrams can be found in Ref. [5]

masses in the theory, we need to use softly broken supersymmetry, in which the couplings are spacetime-independent background superfields. The relation between the RG functions of softly broken and rigid theories was discovered in Refs. [60, 61, 62]. The relation between the correlators of softly broken and rigid theories can be found by a trick of general change of variables in superspace [63].

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Appendix A

$$\begin{aligned}
T_{112} = & -\frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \frac{(yz)_\rho}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} = \\
& + \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}[1y]^{1-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\nu}(z3) \right) \Pi_{\mu\sigma}(z2) \left(\partial_\rho^{(y)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
& + \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}[1y]^{1-\epsilon}} \right) \int Dz \Pi_{\rho\nu}(z3) \Pi_{\mu\sigma}(z2) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
& + \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \Pi_{\mu\sigma}(z2) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \\
& + \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{(1y)_\rho}{[1y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \Pi_{\mu\sigma}(z2) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \equiv \frac{I_1}{2} + \frac{I_2}{2}
\end{aligned}$$

Integral I_2 is $-T_{111}$ and is given by Eq. (7). We do several steps to calculate integral I_1 . Integral I_1 can be done by using the formula (5). Integrals I_1 and I_2 are finite double integrals. The first contribution to I_1 is (with factor $2(1-\epsilon)$)

$$\begin{aligned}
A_1 \equiv & \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{2(yz)_\sigma}{[2z]^{1-\epsilon}[yz]^{2-\epsilon}} = \\
& - \frac{1}{1-\epsilon} \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\sigma^{(y)} \frac{(2y)_\rho}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\sigma^{(y)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}} = \\
& - \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\rho}{[2y]^{2-\epsilon}} \delta(1y) \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[yz]^{1-\epsilon}[2z]^{1-\epsilon}} \\
& + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \delta(2y) \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[yz]^{1-\epsilon}[2z]^{1-\epsilon}} \\
& + \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\rho}{[2y]^{2-\epsilon}[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) k \delta(yz) \frac{1}{[2z]^{1-\epsilon}} = \\
& - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \frac{(21)_\rho}{[12]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[1z][2z]} + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}}
\end{aligned}$$

$$\begin{aligned}
& + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{3-2\epsilon}[1z]^{1-\epsilon}} = \\
& - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \frac{(21)_\rho}{[12]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[1z][2z]} + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{(21)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} \\
& - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{(1z)_\rho}{[2z]^{2-2\epsilon}[1z]^{2-\epsilon}} = \\
& - \frac{k}{2} A_{11} - \frac{k}{4(1-\epsilon)} M_{10} - \frac{k}{8(1-\epsilon)} M_{11}
\end{aligned}$$

Integral A_1 is singular in UV region. Integral A_{11} is finite. Integral M_{10} and and integral M_{11} are defined in Section 3.1.1. The second contribution to I_1 is (with factor 1/2)

$$\begin{aligned}
A_2 \equiv & \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \partial_{(z)}^2 \left(\frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \right) = \\
& - 2(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{(1y)_\rho}{[1y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \partial_{(z)}^2 \left(\frac{(2z)_\sigma}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \right) \\
& + 2(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \int Dz \left(\frac{2g_{\rho\nu}}{[3z]^{1-\epsilon}} - \partial_\nu^{(3)} \frac{(3z)_\rho}{[3z]^{1-\epsilon}} \right) \partial_{(z)}^2 \left(\frac{(2z)_\sigma (yz)_\rho}{[2z]^{1-\epsilon}[yz]^{2-\epsilon}} \right) = \\
& - I_3 - k \frac{(31)_\nu}{[31]^2} \frac{(23)_\sigma}{[23]} \partial_\nu^{(3)} \partial_\sigma^{(2)} J(1, 1, 1) \\
& + 2 \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \int Dz \left(\partial_\rho^{(z)} \frac{1}{[3z]^{1-\epsilon}} \right) \left(\partial_\rho^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} = \\
& - I_3 - k \frac{(31)_\nu}{[31]^2} \frac{(23)_\sigma}{[23]} \partial_\nu^{(3)} \partial_\sigma^{(2)} J(1, 1, 1) \\
& - 4(1-\epsilon) \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy Dz \frac{(2y)_\sigma (2z)_\sigma}{[2y]^{2-\epsilon}[1y]^{1-\epsilon}[3z]^{1-\epsilon}[yz]^{1-\epsilon}[2z]^{2-\epsilon}} \\
& + \frac{k}{2} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \frac{(23)_\sigma}{[23]} \partial_\sigma^{(2)} J(1, 1, 1) - k \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy \frac{1}{[1y]^{1-\epsilon}[3y]^{1-\epsilon}[2y]^{2-2\epsilon}} = \\
& - I_3 - k \frac{(31)_\nu}{[31]^2} \frac{(23)_\sigma}{[23]} \partial_\nu^{(3)} \partial_\sigma^{(2)} J(1, 1, 1) \\
& - \frac{1}{2} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \partial_{(2)}^2 \int Dy Dz \frac{1}{[2y][1y][3z][yz][2z]} \\
& + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{1}{[12]^{1-\epsilon}} \partial_\nu^{(3)} \int Dz \frac{1}{[3z]^{1-\epsilon}[2z]^{2-2\epsilon}} \\
& + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \frac{1}{[23]^{1-\epsilon}} \int Dy Dz \frac{1}{[2y]^{2-2\epsilon}[1y]^{1-\epsilon}} \\
& + \frac{k}{2} \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \frac{(23)_\sigma}{[23]} \partial_\sigma^{(2)} J(1, 1, 1) - k \frac{(31)_\nu}{[31]^2} \partial_\nu^{(3)} \int Dy \frac{1}{[1y]^{1-\epsilon}[3y]^{1-\epsilon}[2y]^{2-2\epsilon}} = \\
& k \left[\frac{1}{[23]^2[31]^2} + \frac{2}{[12][23][31]^2} + \frac{-1}{[12][23]^2[31]} \right]
\end{aligned}$$

$$\begin{aligned}
& +k \left[\frac{-1/2[12]}{[23]^2[31]^2} + \frac{1}{[23][31]^2} + \frac{1/2}{[23]^2[31]} \right] J(1, 1, 1) \\
& +k \left[\frac{1}{[23]^2[31]^2} + \frac{1/2}{[12][23][31]^2} + \frac{1/2}{[12][23]^2[31]} \right] \ln [12] \\
& +k \left[\frac{-1}{[12]^2[31]^2} + \frac{-1}{[23]^2[31]^2} + \frac{1}{[12][23]^2[31]} + \frac{1}{[12]^2[23][31]} \right] \ln [23] \\
& +k \left[\frac{1}{[12]^2[31]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{-3/2}{[12][23]^2[31]^2} + \frac{-1}{[12]^2[23][31]} \right] \ln [31] \\
& + \left[\frac{2}{[12]^2[23]^2} + \frac{2}{[12]^2[31]^2} + \frac{2}{[23]^2[31]^2} + \frac{4}{[12][23][31]^2} + \frac{-4}{[12][23]^2[31]} + \frac{-4}{[12]^2[23][31]} \right] \\
& + \left[\frac{-4}{[12][23]^2} + \frac{2}{[12][31]^2} + \frac{-2[12]}{[23]^2[31]^2} + \frac{4}{[23][31]^2} + \frac{6}{[23]^2[31]} \right] J(1, 1, 1) \\
& + \left[\frac{-2}{[23]^2[31]^2} + \frac{-2}{[12][23][31]^2} + \frac{4}{[12][23]^2[31]} \right] \ln [12] \\
& + \left[\frac{-2}{[12]^2[23]^2} + \frac{-2}{[23]^2[31]^2} + \frac{2}{[12][23][31]^2} + \frac{4}{[12][23]^2[31]} + \frac{2}{[12]^2[23][31]} \right] \ln [23] \\
& + \left[\frac{2}{[12]^2[23]^2} + \frac{4}{[23]^2[31]^2} + \frac{-8}{[12][23]^2[31]} + \frac{-2}{[12]^2[23][31]} \right] \ln [31]
\end{aligned}$$

Integral A_2 is finite in UV and IR. Integral $I_3 = J_2$. The third contribution to I_1 is (with factor $-1/2$)

$$\begin{aligned}
A_3 \equiv & \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_z^2 \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} = \\
& -2 \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\sigma^{(2)} \frac{(2y)_\rho}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\sigma^{(2)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2y)_\rho}{[2y]^2[1y][2z][yz]} \\
& - \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \partial_\rho^{(2)} \delta(2y) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \\
& + \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\rho}{[2y]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) k \delta(2z) \frac{1}{[yz]^{1-\epsilon}} = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2y)_\rho}{[2y]^2[1y][2z][yz]} \\
& - k \frac{(31)_\nu}{[31]^2} \frac{(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} + k \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{(2y)_\rho}{[1y]^{1-\epsilon}[2y]^{3-2\epsilon}} = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2y)_\rho}{[2y]^2[1y][2z][yz]} \\
& - k \frac{(31)_\nu}{[31]^2} \frac{(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{(1y)_\rho}{[1y]^{2-\epsilon}[2y]^{2-2\epsilon}} \equiv \\
& \equiv -A_{31} - \frac{k}{2} M_{10} - \frac{k}{4} M_{11}
\end{aligned}$$

Integral A_3 is singular in UV. Integral A_{31} is finite. The fourth contribution to I_1 is (with factor $-1/2$)

$$\begin{aligned}
A_4 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\sigma}{[2z]^{1-\epsilon}} \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) = \\
&\quad k \frac{(31)_\nu}{[31]^2} \int Dy \left(\partial_\rho^{(y)} \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \Pi_{\rho\nu}(y3) \frac{(2y)_\sigma}{[2y]^{1-\epsilon}} = \\
&\quad k \frac{(31)_\nu}{[31]^2} \int Dy \left(-\frac{g_{\sigma\rho}}{[2y]^{2-\epsilon}} + 2(2-\epsilon) \frac{(2y)_\rho (2y)_\sigma}{[2y]^{3-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} \Pi_{\rho\nu}(y3) \frac{(2y)_\sigma}{[2y]^{1-\epsilon}} = \\
&\quad k(3-2\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\rho\nu}(y3) \frac{(2y)_\rho}{[2y]^{3-2\epsilon} [1y]^{1-\epsilon}} = \\
&\quad -k \frac{3-2\epsilon}{2} \frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\rho\nu}(y3) \frac{(1y)_\rho}{[2y]^{2-2\epsilon} [1y]^{2-\epsilon}} = -k \frac{3-2\epsilon}{4} M_{11}
\end{aligned}$$

This integral is singular. We collect all singular integrals from A_1 , A_3 and A_4 .

$$\begin{aligned}
M_2 &\equiv -\frac{k}{2} M_{10} - \frac{k}{4} M_{11} + \frac{k}{4} M_{10} + \frac{k}{8} M_{12} + k \frac{3-2\epsilon}{8} M_{11} = \\
&\quad -\frac{k}{4} M_{10} + \frac{k}{8} M_{12} + k \frac{1-2\epsilon}{8} M_{11} = \\
&\quad k \left[\frac{1/8}{[12]^2 [23]^2} + \frac{-5/8}{[12]^2 [31]^2} + \frac{1/8}{[12]^2 [23]^2} + \frac{1/2}{[12] [23] [31]^2} + \frac{-1/4}{[12] [23]^2 [31]} + \frac{1/2}{[12]^2 [23] [31]} \right] \\
&\quad + k \left[\frac{-1/4}{[12]^2 [23]^2} + \frac{1/2}{[12]^2 [31]^2} + \frac{-1/4}{[23]^2 [31]^2} + \frac{-1/4}{[12] [23] [31]^2} + \frac{1/2}{[12] [23]^2 [31]} + \frac{-1/4}{[12]^2 [23] [31]} \right] \ln[12] \\
&\quad + k \left[\frac{1/8}{[12]^2 [23]^2} + \frac{-1/4}{[12]^2 [31]^2} + \frac{1/8}{[23]^2 [31]^2} + \frac{1/8}{[12] [23] [31]^2} + \frac{-1/4}{[12] [23]^2 [31]} + \frac{1/8}{[12]^2 [23] [31]} \right] \ln[23] \\
&\quad + k \left[\frac{1/8}{[12]^2 [23]^2} + \frac{-1/4}{[12]^2 [31]^2} + \frac{1/8}{[23]^2 [31]^2} + \frac{1/8}{[12] [23] [31]^2} + \frac{-1/4}{[12] [23]^2 [31]} + \frac{1/8}{[12]^2 [23] [31]} \right] \ln[31]
\end{aligned}$$

Combining two finite integrals from A_1 and A_3 , we obtain

$$\begin{aligned}
K_2 &\equiv -k A_{11} + \frac{1}{2} A_{31} = k \left[\frac{-1/2}{[12]^2 [31]^2} + \frac{-1/2}{[12] [23] [31]^2} + \frac{1/2}{[12]^2 [23] [31]} \right] \\
&\quad + k \left[\frac{-1/2}{[12] [31]^2} + \frac{[23]}{[12]^2 [31]^2} + \frac{-1}{[12]^2 [31]} \right] J(1, 1, 1) \\
&\quad + k \left[\frac{-1/2}{[12]^2 [31]^2} + \frac{1/2}{[12]^2 [23] [31]} \right] \ln[12] + k \left[\frac{1/2}{[12]^2 [31]^2} + \frac{-1/2}{[12] [23] [31]^2} \right] \ln[31] \\
&\quad + \left[\frac{1}{[12]^2 [23]^2} + \frac{-1}{[12] [23]^2 [31]} + \frac{-1}{[12]^2 [23] [31]} \right] \\
&\quad + \left[\frac{1}{[12] [23]^2} + \frac{1}{[12]^2 [31]} + \frac{-[31]}{[12]^2 [23]^2} \right] J(1, 1, 1) \\
&\quad + \left[\frac{1}{[12]^2 [23]^2} + \frac{-2}{[12] [23]^2 [31]} + \frac{-1}{[12]^2 [23] [31]} \right] \ln[12] \\
&\quad + \left[\frac{-1}{[12]^2 [23]^2} + \frac{1}{[12] [23]^2 [31]} + \frac{1}{[12]^2 [23] [31]} \right] \ln[23] + \frac{1}{[12] [23]^2 [31]} \ln[31]
\end{aligned}$$

Collecting all the parts together,

$$I_1 \equiv K_2 + \frac{1}{2}A_2 + M_2$$

The result is written in Eq. (8).

Appendix B

In this Appendix we provide the calculation of T_{21} . We will reduce the number of indices further by means of simple algebra,

$$\begin{aligned} \Pi_{\rho\sigma}(2z) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} &= \left(\frac{g_{\rho\sigma}}{[2z]^{1-\epsilon}} + 2(1-\epsilon) \frac{(2z)_\rho(2z)_\sigma}{[2z]^{2-\epsilon}} \right) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} = \\ &= \left(\frac{2g_{\rho\sigma}}{[2z]^{1-\epsilon}} - \partial_\sigma^{(2)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} = \frac{2(2y)_\rho}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} - \left(\partial_\sigma^{(2)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} = \\ &= \frac{2(2y)_\rho}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} + \frac{1}{2(1-\epsilon)} \left(\partial_\sigma^{(2)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \left(\partial_\sigma^{(2)} \frac{1}{[2y]^{1-\epsilon}} \right) = \\ &= \frac{2(2y)_\rho}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} + \frac{1}{4(1-\epsilon)} \left[\partial_{(2)}^2 \left(\frac{(2z)_\rho}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}} \right) - \left(\partial_{(2)}^2 \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \right. \\ &\quad \left. - \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \right] \end{aligned} \quad (\text{B.1})$$

We need to calculate the following integral

$$\begin{aligned} T_{21} &= \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \times \\ &\quad \times \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} = \\ &= \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \left\{ \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \right\} \end{aligned}$$

Now we take into account the following equality,

$$\begin{aligned} \frac{2(1y)_\mu}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} - \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} &= \\ \frac{1}{1-\epsilon} \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) + \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \end{aligned} \quad (\text{B.2})$$

The first term on the l.h.s. of Eq. (B.2) corresponds to the finite part of T_{21} , while the second term on the l.h.s. of Eq. (B.2) contains a singular contribution. On the r.h.s. we produce a mixture of singular and finite contributions. However, it is more easy to work with such a representation. We do several steps to calculate integral T_{21} , taking into account Eq. (B.1). The first contribution (with factor $1/(1-\epsilon)$) to T_{21} is

$$B_1 \equiv \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{2(2y)_\rho}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} \right) \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) =$$

$$\begin{aligned}
& \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{2(2y)_\mu}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} \right) \partial_\rho^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& \frac{1}{1-\epsilon} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \left(\partial_\mu^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \partial_\rho^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left[\partial_{(2)}^2 \left(\frac{1}{[2y]^{1-\epsilon}[2z]^{1-\epsilon}} \right) \right. \\
& \left. - \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}} - \frac{1}{[2y]^{1-\epsilon}} \left(\partial_{(2)}^2 \frac{1}{[2z]^{1-\epsilon}} \right) \right] \partial_\rho^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[2z]^{1-\epsilon}} \partial_\rho^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) \\
& - \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) k\delta(2y) \frac{1}{[2z]^{1-\epsilon}} \partial_\rho^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) \\
& - \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) k\delta(2z) \frac{1}{[2y]^{1-\epsilon}} \partial_\rho^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[2z]^{1-\epsilon}} \frac{(zy)_\rho}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \\
& + \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[2z]^{1-\epsilon}} \frac{(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} \\
& - \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{1-\epsilon}} \partial_\rho^{(2)} \left(\frac{1}{[2z]^{1-\epsilon}[12]^{1-\epsilon}} \right) \\
& - \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{1}{[2y]^{1-\epsilon}} \partial_\rho^{(y)} \left(\frac{1}{[y2]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(yz)_\rho}{[yz]^2[1y][2y][2z]} \\
& + \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(1y)_\rho}{[yz][1y]^2[2y][2z]} \\
& + k \frac{(31)_\nu}{[31]^2} \frac{(21)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} + k \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{(2y)_\rho}{[2y]^{3-2\epsilon}[1y]^{1-\epsilon}} = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(yz)_\rho}{[yz]^2[1y][2y][2z]} \\
& + \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(1y)_\rho}{[yz][1y]^2[2y][2z]} \\
& + k \frac{(31)_\nu}{[31]^2} \frac{(21)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{(1y)_\rho}{[2y]^{2-2\epsilon}[1y]^{2-\epsilon}} \equiv \\
& \equiv -B_{11} + J_{31}/2 - \frac{k}{2} M_{10} - \frac{k}{4} M_{12}
\end{aligned}$$

Integral B_1 is singular in UV. Integral B_{11} is finite. We realize the strategy to present all the integrals in terms of basic integrals used to calculate T_1 . The second contribution to T_{21} (with

factor 1) is

$$\begin{aligned}
B_2 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{2(2y)_\rho}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} \right) \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} = \\
&\frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}} \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} = \\
&\frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}} \left[\partial_{(z)}^2 \left(\frac{1}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}} \right) \right. \\
&\quad \left. - \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}} - \frac{1}{[yz]^{1-\epsilon}} \left(\partial_{(z)}^2 \frac{1}{[2z]^{1-\epsilon}} \right) \right] \frac{1}{[1y]^{1-\epsilon}} = \\
&\frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}[2z]^{1-\epsilon}[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \\
&\quad - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}} \delta(yz) \frac{1}{[2z]^{1-\epsilon}[1y]^{1-\epsilon}} \\
&\quad - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}[yz]^{1-\epsilon}} \delta(2z) \frac{1}{[1y]^{1-\epsilon}} = \\
&\frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}[2z]^{1-\epsilon}[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \\
&\quad - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{2(2z)_\rho}{[2z]^{3-2\epsilon}[1z]^{1-\epsilon}} - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\rho\nu}(23) \frac{2(2y)_\rho}{[2y]^{3-2\epsilon}[1y]^{1-\epsilon}} = \\
&\quad \frac{1}{2} \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2y)_\rho}{[2y]^2[2z][yz][1y]} \\
&\quad + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{(1z)_\rho}{[2z]^{2-2\epsilon}[1z]^{2-\epsilon}} + \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{(1y)_\rho}{[2y]^{2-2\epsilon}[1y]^{2-\epsilon}} \equiv \\
&\quad \equiv B_{21}/2 + \frac{k}{8(1-\epsilon)} M_{11} + \frac{k}{8(1-\epsilon)} M_{12}
\end{aligned}$$

Integral B_2 is singular in UV. Integral B_{21} is finite. M -integrals are singular. The third contribution to T_{21} corresponds to the second term in Eq. (B.1) (with factor $(1/4)/(1-\epsilon)^2$) is

$$\begin{aligned}
B_3 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \partial_{(2)}^2 \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}} \right) \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
&\frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}} \right) \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
&\frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(y)} \frac{(2z)_\mu}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}} \right) \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
&2(1-\epsilon) \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\mu(2y)_\mu}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
&2 \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\mu(2y)_\mu}{[2z][2y]^2} \partial_\rho^{(z)} \left(\frac{1}{[yz][1y]} \right)
\end{aligned}$$

Integral B_3 is finite integral in UV and IR. Another contribution to T_{21} that corresponds to the second term in Eq. (B.1) is (with factor $(1/4)/(1-\epsilon)$)

$$\begin{aligned}
B_4 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \partial_{(2)}^2 \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon} [2y]^{1-\epsilon}} \right) \frac{(yz)_\mu}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
&\frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}} \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \left(\partial_\mu^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} = \\
&\frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}} \left[\partial_{(z)}^2 \left(\frac{(2z)_\rho}{[2z]^{1-\epsilon}} \frac{1}{[yz]^{1-\epsilon}} \right) \right. \\
&\quad \left. - \left(\partial_{(z)}^2 \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} - \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right] \frac{1}{[1y]^{1-\epsilon}} = \\
&\frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{1-\epsilon} [yz]^{1-\epsilon} [1y]^{1-\epsilon} [2y]^{1-\epsilon}} \\
&\quad - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}} \frac{1}{[2z]^{1-\epsilon}} \frac{(yz)_\rho}{[yz]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \\
&\quad - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}} \delta(yz) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} = \\
&\frac{1}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{1-\epsilon} [yz]^{1-\epsilon} [1y]^{1-\epsilon} [2y]^{1-\epsilon}} \\
&\quad - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon} [2y]^{1-\epsilon} [2z]^{1-\epsilon}} \\
&\quad - \frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{2-2\epsilon} [1z]^{1-\epsilon}} = \\
&\quad \frac{1}{4} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z][yz][1y][2y]} \\
&- \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(yz)_\rho}{[yz]^2 [1y][2y][2z]} - \frac{k}{4} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^2} \frac{1}{[1z]} \equiv \\
&\equiv \frac{1}{4} B_{41} - B_{11} - \frac{k}{4} B_{42}.
\end{aligned}$$

Integral B_4 is finite in UV and IR. Integrals B_{41} , B_{11} and B_{42} are finite. A contribution to T_{21} that corresponds to the third term in Eq. (B.1) is (with factor $-(1/4)/(1-\epsilon)^2$)

$$\begin{aligned}
B_5 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_{(2)}^2 \partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon} [1y]^{1-\epsilon}} \right) = \\
&-4(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon} [1y]^{1-\epsilon}} \right) = \\
&-8(1-\epsilon)^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \frac{(2y)_\mu}{[2y]^{2-\epsilon}} \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon} [1y]^{1-\epsilon}} \right) = \\
&\quad - \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left[\partial_{(2)}^2 \left(\frac{1}{[2z]^{1-\epsilon} [2y]^{1-\epsilon}} \right) \right]
\end{aligned}$$

$$\begin{aligned}
& - \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}} - \frac{1}{[2y]^{1-\epsilon}} \left(\partial_{(2)}^2 \frac{1}{[2z]^{1-\epsilon}} \right) \Big] \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}} \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) \\
& + \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) k \delta(2y) \frac{1}{[2z]^{1-\epsilon}} \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) \\
& + \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}} k \delta(2z) \partial_\rho^{(z)} \left(\frac{1}{[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& + 2(1-\epsilon) \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{2-\epsilon}[2y]^{1-\epsilon}[yz]^{1-\epsilon}[1y]^{1-\epsilon}} \\
& + k \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{1-\epsilon}} \partial_\rho^{(z)} \left(\frac{1}{[2z]^{1-\epsilon}[12]^{1-\epsilon}} \right) \\
& + k \frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\rho\nu}(23) \frac{1}{[2y]^{1-\epsilon}} \partial_\rho^{(2)} \left(\frac{1}{[y2]^{1-\epsilon}[1y]^{1-\epsilon}} \right) = \\
& + 2 \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^2[2y][yz][1y]} \\
& - 2k(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\rho\nu}(23) \frac{(2y)_\rho}{[2y]^{3-2\epsilon}[1y]^{1-\epsilon}} = \\
& + 2 \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^2[2y][yz][1y]} \\
& + k(1-\epsilon) \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dy \frac{(1y)_\rho}{[2y]^{2-2\epsilon}[1y]^{2-\epsilon}} \equiv 2B_{51} + \frac{k(1-\epsilon)}{2} M_{12}.
\end{aligned}$$

Integral B_{51} is finite. Another contribution to T_{21} that corresponds to the third term in Eq. (B.1) is (with factor $-(1/4)/(1-\epsilon)$)

$$\begin{aligned}
B_6 & \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_{(2)}^2 \partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} = \\
& - 4(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \frac{(yz)_\mu}{[yz]^{2-\epsilon}[1y]^{1-\epsilon}} = \\
& - \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} \left[\partial_{(z)}^2 \left(\frac{(2z)_\rho}{[2z]^{2-\epsilon}} \frac{1}{[yz]^{1-\epsilon}} \right) \right. \\
& \quad \left. - \left(\partial_{(z)}^2 \frac{(2z)_\rho}{[2z]^{2-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} - \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \frac{(2z)_\rho}{[2z]^{2-\epsilon}} \right] = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{2-\epsilon}[yz]^{1-\epsilon}[2y]^{1-\epsilon}[1y]^{1-\epsilon}} \\
& - \frac{1}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} k \partial_\rho^{(2)} \delta(2z) \frac{1}{[yz]^{1-\epsilon}} \\
& + \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} k \delta(yz) \frac{(2z)_\rho}{[2z]^{2-\epsilon}} =
\end{aligned}$$

$$\begin{aligned}
& -\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{2-\epsilon} [yz]^{1-\epsilon} [2y]^{1-\epsilon} [1y]^{1-\epsilon}} \\
& -k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \delta(2z) \frac{(yz)_\rho}{[yz]^{2-\epsilon}} + k \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{3-2\epsilon} [1z]^{1-\epsilon}} = \\
& -\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^2 [yz] [2y] [1y]} \\
& -k \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dz \frac{(y2)_\rho}{[2y]^{3-2\epsilon} [1y]^{1-\epsilon}} + k \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^{3-2\epsilon} [1z]^{1-\epsilon}} = \\
& -\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\rho\nu}(z3) \frac{(2z)_\rho}{[2z]^2 [yz] [2y] [1y]} \\
& -\frac{k}{2} \frac{(31)_\nu}{[31]^2} \Pi_{\rho\nu}(23) \int Dz \frac{(1y)_\rho}{[2y]^{2-2\epsilon} [1y]^{2-\epsilon}} - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \frac{(1z)_\rho}{[2z]^{2-2\epsilon} [1z]^{2-\epsilon}} \equiv \\
& \equiv -B_{61} - \frac{k}{4} M_{12} - \frac{k}{4} M_{11}
\end{aligned}$$

Integral B_{61} is finite. Another contribution to T_{21} that corresponds to the fourth term in Eq. (B.1) is (with factor $-(1/4)/((1-\epsilon)^2)$)

$$\begin{aligned}
B_7 & \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon} [1y]^{1-\epsilon}} \right) = \\
& \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) k \delta(2y) \partial_\mu^{(y)} \left(\frac{1}{[yz]^{1-\epsilon} [1y]^{1-\epsilon}} \right) = \\
& k \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \partial_\mu^{(2)} \left(\frac{1}{[2z]^{1-\epsilon} [12]^{1-\epsilon}} \right) = \\
& -k(1-\epsilon) \frac{(31)_\nu}{[31]^2} \frac{(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} = -\frac{k(1-\epsilon)}{2} M_{10}
\end{aligned}$$

Another contribution to T_{21} that corresponds to the fourth term in Eq. (B.1) is (with factor $-(1/4)/(1-\epsilon)$)

$$\begin{aligned}
& \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{(yz)_\mu}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
& \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) k \delta(2y) \frac{(yz)_\mu}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\rho\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{(2z)_\mu}{[2z]^{2-\epsilon} [12]^{1-\epsilon}} = 0
\end{aligned}$$

This integral is zero due to transversality of the gluon propagator. Combining the singular contribution T_{23} from Eq. (13) with the singular single integrals from B_1 , B_2 , B_5 , B_6 and B_7 we obtain

$$\begin{aligned}
M_3 & \equiv \frac{3k}{8(1-\epsilon)} M_{10} - \frac{k}{2(1-\epsilon)} M_{10} - \frac{k}{4(1-\epsilon)} M_{12} + \frac{k}{8(1-\epsilon)} M_{11} + \frac{k}{8(1-\epsilon)} M_{12} \\
& - \frac{k}{8(1-\epsilon)} M_{12} + \frac{k}{16(1-\epsilon)} M_{12} + \frac{k}{16(1-\epsilon)} M_{11} + \frac{k}{8(1-\epsilon)} M_{10} =
\end{aligned}$$

$$\begin{aligned}
& k \left[\frac{3/16}{[12]^2[23]^2} + \frac{3/16}{[12]^2[31]^2} + \frac{3/16}{[23]^2[31]^2} + \frac{-3/8}{[12][23][31]^2} + \frac{-3/8}{[12][23]^2[31]} + \frac{-3/8}{[12]^2[23][31]} \right] \\
& + k \left[\frac{-3/16}{[12]^2[23]^2} + \frac{3/8}{[12]^2[31]^2} + \frac{-3/16}{[23]^2[31]^2} + \frac{-3/16}{[12][23][31]^2} + \frac{3/8}{[12][23]^2[31]} + \frac{-3/16}{[12]^2[23][31]} \right] \ln[23] \\
& + k \left[\frac{3/16}{[12]^2[23]^2} + \frac{-3/8}{[12]^2[31]^2} + \frac{3/16}{[23]^2[31]^2} + \frac{3/16}{[12][23][31]^2} + \frac{-3/8}{[12][23]^2[31]} + \frac{3/16}{[12]^2[23][31]} \right] \ln[31]
\end{aligned}$$

Combining finite contributions from finite double integrals and single finite integral of B_1, \dots, B_7 we obtain

$$\begin{aligned}
K_3 \equiv & -B_{11} + J_{31}/2 + B_{21}/2 + B_3/4 + B_{41}/16 - B_{11}/4 - kB_{42}/16 - B_{51}/2 + B_{61}/4 = \\
& k \left[\frac{1/8}{[12]^2[23]^2} + \frac{-1/4}{[12]^2[31]^2} + \frac{-1/8}{[23]^2[31]^2} + \frac{3/8}{[12][23][31]^2} + \frac{1/8}{[12]^2[23][31]} \right] \\
& + k \left[\frac{1/4[12]}{[23]^2[31]^2} + \frac{-1/4}{[23][31]^2} + \frac{-1/4}{[23]^2[31]} \right] J(1, 1, 1) + k \frac{1/4}{[23]^2[31]^2} \ln[12] \\
& + k \left[\frac{1/4}{[23]^2[31]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{-1/4}{[12][23]^2[31]} \right] \ln[23] \\
& + k \left[\frac{-1/2}{[23]^2[31]^2} + \frac{1/2}{[12][23][31]^2} + \frac{1/4}{[12][23]^2[31]} \right] \ln[31] \\
& + \left[\frac{1/2}{[12]^2[23]^2} + \frac{-1}{[23]^2[31]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{3/2}{[12]^2[23][31]} \right] \\
& + \left[\frac{-3}{[12][23]^2} + \frac{1}{[12][31]^2} + \frac{1/2[12]}{[23]^2[31]^2} + \frac{-1}{[23][31]^2} + \frac{-2}{[12]^2[31]} + \frac{3/2}{[23]^2[31]} + \frac{[31]}{[12]^2[23]^2} \right] J(1, 1, 1) \\
& + \left[\frac{-2}{[12]^2[23]^2} + \frac{-1/2}{[12][23][31]^2} + \frac{5/2}{[12][23]^2[31]} + \frac{1}{[12]^2[23][31]} \right] \ln[12] \\
& + \left[\frac{-1/2}{[12]^2[23]^2} + \frac{1}{[23]^2[31]^2} + \frac{-1}{[12][23][31]^2} + \frac{-1/2}{[12][23]^2[31]} + \frac{-5/2}{[12]^2[23][31]} \right] \ln[23] \\
& + \left[\frac{5/2}{[12]^2[23]^2} + \frac{-1}{[23]^2[31]^2} + \frac{3/2}{[12][23][31]^2} + \frac{-2}{[12][23]^2[31]} + \frac{3/2}{[12]^2[23][31]} \right] \ln[31]
\end{aligned}$$

Thus,

$$T_{21} + T_{23} = K_3 + M_3$$

Appendix C

In this Appendix integral T_{311} is calculated,

$$\begin{aligned}
T_{311} = & \frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} = \\
& \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}}
\end{aligned}$$

According to Eq. (B.1) the first contribution to T_{311} is (with factor 1)

$$\begin{aligned}
C_1 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{2(2y)_\rho}{[2z]^{1-\epsilon}[2y]^{2-\epsilon}} \right) \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} = \\
&\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}} \frac{2(1y)_\rho}{[1y]^{2-\epsilon}} \frac{1}{[yz]^{1-\epsilon}} = \\
&\frac{1}{(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \left(\partial_\rho^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \left(\partial_\rho^{(y)} \frac{1}{[1y]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} = \\
&\frac{1}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \left[\partial_{(y)}^2 \left(\frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} \right) \right. \\
&\quad \left. - \left(\partial_{(y)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} - \frac{1}{[2y]^{1-\epsilon}} \left(\partial_{(y)}^2 \frac{1}{[1y]^{1-\epsilon}} \right) \right] \frac{1}{[yz]^{1-\epsilon}} = \\
&\frac{1}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} k\delta(yz) \\
&- \frac{1}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}[yz]^{1-\epsilon}} k\delta(1y) \\
&- \frac{1}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}[yz]^{1-\epsilon}} k\delta(2y) = \\
&\frac{k}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}[1z]^{1-\epsilon}} \\
&- \frac{k}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[12]^{1-\epsilon}[1z]^{1-\epsilon}} \\
&- \frac{k}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[12]^{1-\epsilon}[2z]^{1-\epsilon}} = \\
&\frac{k}{1-\epsilon} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{3-2\epsilon}[1z]^{1-\epsilon}} - k \frac{(31)_\nu}{[31]^2[12]} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[1z]} = \\
&- \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(1z)_\mu}{[2z]^{2-2\epsilon}[1z]^{2-\epsilon}} - k \frac{(31)_\nu}{[31]^2[12]} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[1z]} \equiv \\
&\equiv -\frac{k}{4(1-\epsilon)} M_{11} - kC_{11}
\end{aligned}$$

Integral C_{11} is finite. The second contribution to T_{311} (with factor $1/(4(1-\epsilon))$) is

$$\begin{aligned}
C_2 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \partial_{(2)}^2 \left(\frac{(2z)_\rho}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}} \right) \right) \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} = \\
&\frac{1}{1-\epsilon} \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \frac{1}{[yz]^{1-\epsilon}} \left(\partial_\rho^{(y)} \frac{1}{[1y]^{1-\epsilon}} \right) = \\
&- \frac{1}{1-\epsilon} \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \right) \left(\partial_\rho^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \\
&- \frac{1}{1-\epsilon} \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \left(\partial_\rho^{(y)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} = \\
&2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \left(\partial_\rho^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \frac{(yz)_\mu}{[yz]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}}
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{1-\epsilon}\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\left(\partial_\rho^{(z)}\frac{(2z)_\mu}{[2z]^{1-\epsilon}}\right)\frac{1}{[2y]^{1-\epsilon}}\left(\partial_\rho^{(y)}\frac{1}{[yz]^{1-\epsilon}}\right)\frac{1}{[1y]^{1-\epsilon}}= \\
& \quad 4(1-\epsilon)\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\rho}{[2z]^{1-\epsilon}}\frac{(2y)_\rho}{[2y]^{2-\epsilon}}\frac{(yz)_\mu}{[yz]^{2-\epsilon}}\frac{1}{[1y]^{1-\epsilon}} \\
& \quad +\frac{1}{1-\epsilon}\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\left(\partial_\rho^{(z)}\frac{(2z)_\mu}{[2z]^{1-\epsilon}}\right)\left(\partial_\rho^{(z)}\frac{1}{[yz]^{1-\epsilon}}\right)\frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}}= \\
& \quad 2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{[2z]+[2y]-[yz]}{[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}}\frac{(yz)_\mu}{[yz]^2[1y]} \\
& \quad +\frac{1}{2(1-\epsilon)}\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\left[\partial_{(z)}^2\left(\frac{(2z)_\mu}{[2z]^{1-\epsilon}[yz]^{1-\epsilon}}\right)+4(1-\epsilon)\frac{(2z)_\mu}{[2z]^{2-\epsilon}[yz]^{1-\epsilon}}\right. \\
& \quad \quad \left.-\frac{(2z)_\mu}{[2z]^{1-\epsilon}}k\delta(yz)\right]\frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}}= \\
& \quad 2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(yz)_\mu}{[2z]^{-2\epsilon}[2y]^{2-2\epsilon}[yz]^2[1y]} \\
& \quad +2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& \quad -2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(yz)_\mu}{[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}[yz][1y]} \\
& \quad +\frac{1}{2}\frac{(31)_\nu}{[31]^2}\partial_{(2)}^2\partial_{(3)}^2\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z][yz][2y][1y]} \\
& \quad +2\frac{(31)_\nu}{[31]^2}\partial_{(2)}^2\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^2[yz][2y][1y]}-\frac{k}{2}\frac{(31)_\nu}{[31]^2}\partial_{(2)}^2\int Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^2[1z]}= \\
& \quad 4\epsilon\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}[yz][1y]} \\
& \quad +2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& \quad +2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2y)_\mu}{[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}[yz][1y]} \\
& \quad -2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}[yz][1y]} \\
& \quad +\frac{1}{2}\frac{(31)_\nu}{[31]^2}\partial_{(2)}^2\partial_{(3)}^2\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z][yz][2y][1y]} \\
& \quad +2\frac{(31)_\nu}{[31]^2}\partial_{(2)}^2\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^2[yz][2y][1y]}-\frac{k}{2}\frac{(31)_\nu}{[31]^2}\partial_{(2)}^2\int Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^2[1z]}= \\
& \quad -\frac{(2-4\epsilon)A(1-2\epsilon,1,1)}{\epsilon(1-2\epsilon)}\partial_{(2)}^2\frac{(31)_\nu}{[31]^2[12]^{1-\epsilon}}\int Dz\Pi_{\mu\nu}(z3)\frac{(2z)_\mu}{[2z]^{2-3\epsilon}[1z]^\epsilon} \\
& \quad +2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& \quad +2\partial_{(2)}^2\frac{(31)_\nu}{[31]^2}\int Dy Dz\Pi_{\mu\nu}(z3)\frac{(2y)_\mu}{[2z]^{1-2\epsilon}[2y]^{2-2\epsilon}[yz][1y]}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z][yz][2y][1y]} \\
& + 2 \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[yz][2y][1y]} - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[1z]} = \\
& \quad 2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2[12]} \int Dz \Pi_{\mu\nu}(z3) \frac{(1z)_\mu}{[2z][1z]} \\
& \quad + 2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& \quad + 2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2y)_\mu}{[2z][2y]^2[yz][1y]} \\
& \quad + \frac{1}{2} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z][yz][2y][1y]} \\
& + 2 \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[yz][2y][1y]} - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[1z]} \equiv \\
& \quad \equiv 2C_{21} + 2B_{11} + 2A_{31} + \frac{1}{2}B_{41} + 2B_{51} - \frac{k}{2}B_{42}
\end{aligned}$$

All A -integrals and B -integrals are finite and were calculated in Appendices A and B, respectively. Integral C_{21} is finite. The third contribution to T_{311} is (with factor $-1/(4(1-\epsilon))$)

$$\begin{aligned}
C_3 & \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \left(\partial_{(2)}^2 \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \right) \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} = \\
& -4 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \frac{1}{[yz]^{1-\epsilon}} \left(\partial_\rho^{(y)} \frac{1}{[1y]^{1-\epsilon}} \right) = \\
& \quad 4 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \left(\partial_\rho^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \\
& + 4 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \left(\partial_\rho^{(y)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} = \\
& \quad -8(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\rho}{[2z]^{2-\epsilon}} \left(\partial_\rho^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \frac{(yz)_\mu}{[yz]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \\
& -4 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \left(\partial_\rho^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} = \\
& \quad -4 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(2)} \frac{1}{[2z]^{1-\epsilon}} \right) \left(\partial_\rho^{(2)} \frac{1}{[2y]^{1-\epsilon}} \right) \frac{(yz)_\mu}{[yz]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} \\
& -4 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \left(\partial_\rho^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} = \\
& \quad -2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z]^{1-\epsilon}[2y]^{1-\epsilon}[yz]^{2-\epsilon}[1y]^{1-\epsilon}} \\
& \quad + 2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2y]^{1-\epsilon}[yz]^{2-\epsilon}[1y]^{1-\epsilon}} k\delta(2z)
\end{aligned}$$

$$\begin{aligned}
& +2\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z]^{1-\epsilon}[yz]^{2-\epsilon}[1y]^{1-\epsilon}} k\delta(2y) \\
& -2\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left[\partial_{(z)}^2 \left(\frac{(2z)_\mu}{[2z]^{2-\epsilon}[yz]^{1-\epsilon}} \right) - \frac{k}{2(1-\epsilon)} \partial_\mu^{(z)} \delta(2z) \frac{1}{[yz]^{1-\epsilon}} \right. \\
& \quad \left. - \frac{(2z)_\mu}{[2z]^{2-\epsilon}} k\delta(yz) \right] \frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} = \\
& -2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& +2k\frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\mu\nu}(23) \frac{(y2)_\mu}{[2y]^{1-\epsilon}[2y]^{2-\epsilon}[1y]^{1-\epsilon}} \\
& +2k\frac{(31)_\nu}{[31]^2[12]^{1-\epsilon}} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{3-2\epsilon}} \\
& -2\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{2-\epsilon}[yz]^{1-\epsilon}[2y]^{1-\epsilon}[1y]^{1-\epsilon}} \\
& -2k\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[yz]^{2-\epsilon}[2y]^{1-\epsilon}[1y]^{1-\epsilon}} \delta(2z) \\
& +2k\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \delta(yz) \frac{1}{[2y]^{1-\epsilon}[1y]^{1-\epsilon}} = \\
& -2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} - 2k\frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\mu\nu}(23) \frac{(2y)_\mu}{[2y]^{3-2\epsilon}[1y]^{1-\epsilon}} \\
& -2\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[yz][2y][1y]} + 2k\frac{(31)_\nu}{[31]^2} \int Dy \Pi_{\mu\nu}(23) \frac{(2y)_\mu}{[2y]^{3-2\epsilon}[1y]^{1-\epsilon}} \\
& +2k\frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{3-2\epsilon}[1z]^{1-\epsilon}} = \\
& -2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& -2\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[yz][2y][1y]} + 2k\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{3-2\epsilon}[1z]^{1-\epsilon}} = \\
& -2\partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(yz)_\mu}{[2z][2y][yz]^2[1y]} \\
& -2\frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[yz][2y][1y]} - k\frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(1z)_\mu}{[2z]^{2-2\epsilon}[1z]^{2-\epsilon}} \equiv \\
& \equiv -2B_{11} - 2B_{61} - \frac{k}{2}M_{11}.
\end{aligned}$$

The fourth contribution to T_{311} is (with factor $-1/(4(1-\epsilon))$).

$$C_4 \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{2(1y)_\rho}{[yz]^{1-\epsilon}[1y]^{2-\epsilon}} =$$

$$\begin{aligned}
& k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \delta(2y) \frac{2(1y)_\rho}{[yz]^{1-\epsilon} [1y]^{2-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{2(12)_\rho}{[2z]^{1-\epsilon} [12]^{2-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[2z]^{1-\epsilon}} = \\
& -2(1-\epsilon) k \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\rho (2z)_\mu}{[2z]^{3-2\epsilon}} = \\
& -\frac{k}{2} \frac{(31)_\nu}{[31]^2} \frac{2(12)_\rho}{[12]^{2-\epsilon}} \int Dz \Pi_{\rho\nu}(z3) \frac{1}{[2z]^{2-2\epsilon}} = -\frac{k}{2} M_{10}
\end{aligned}$$

The sum of the singular integrals from contribution C_1 , C_3 , and C_4 is

$$\begin{aligned}
M_4 & \equiv -\frac{k}{4(1-\epsilon)} M_{11} + \frac{k}{8(1-\epsilon)} M_{11} + \frac{k}{8(1-\epsilon)} M_{10} = \\
& k \left[\frac{1/8}{[12]^2 [23]^2} + \frac{-1/4}{[12]^2 [31]^2} + \frac{1/8}{[23]^2 [31]^2} + \frac{1/8}{[12][23][31]^2} + \frac{-1/4}{[12][23]^2 [31]} + \frac{1/8}{[12]^2 [23][31]} \right] \ln[12] \\
& + k \left[\frac{-1/8}{[12]^2 [23]^2} + \frac{1/4}{[12]^2 [31]^2} + \frac{-1/8}{[23]^2 [31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{1/4}{[12][23]^2 [31]} + \frac{-1/8}{[12]^2 [23][31]} \right] \ln[31]
\end{aligned}$$

The sum of the finite integrals from C_1 , C_2 , C_3 , and C_4 is

$$\begin{aligned}
K_4 & \equiv -kC_{11} + \frac{1}{2}C_{21} + \frac{1}{2}B_{11} + \frac{1}{2}A_{31} + \frac{1}{8}B_{41} + \frac{1}{2}B_{51} - \frac{k}{8}B_{42} + \frac{1}{2}B_{11} + \frac{1}{2}B_{61} = \\
& k \left[\frac{1/4}{[12]^2 [23]^2} + \frac{-1/4}{[23]^2 [31]^2} + \frac{5/4}{[12][23][31]^2} + \frac{-1/4}{[12]^2 [23][31]} \right] \\
& + k \left[\frac{-1/2}{[12][31]^2} + \frac{1/2[12]}{[23]^2 [31]^2} + \frac{-1/2}{[23][31]^2} + \frac{-1/2}{[23]^2 [31]} \right] J(1, 1, 1) \\
& + k \left[\frac{-1/2}{[23]^2 [31]^2} + \frac{-1/2}{[12][23][31]^2} \right] \ln[12] + k \left[\frac{1}{[12]^2 [31]^2} + \frac{1/2}{[23]^2 [31]^2} + \frac{-1/2}{[12][23]^2 [31]} \right] \ln[23] \\
& + k \left[\frac{-1}{[12]^2 [31]^2} + \frac{1/2}{[12][23][31]^2} + \frac{1/2}{[12][23]^2 [31]} \right] \ln[31] \\
& + \left[\frac{2}{[12]^2 [23]^2} + \frac{2}{[12][23][31]^2} + \frac{-2}{[12][23]^2 [31]} \right] \\
& + \left[\frac{1}{[12][23]^2} + \frac{[12]}{[23]^2 [31]^2} + \frac{-2}{[23][31]^2} + \frac{-1}{[23]^2 [31]} + \frac{-[31]}{[12]^2 [23]^2} \right] J(1, 1, 1) \\
& + \left[\frac{1}{[12][23][31]^2} + \frac{-2}{[12][23]^2 [31]} + \frac{-1}{[12]^2 [23][31]} \right] \ln[12] + \left[\frac{-2}{[12]^2 [23]^2} + \frac{2}{[12][23]^2 [31]} \right] \ln[23] \\
& + \left[\frac{2}{[12]^2 [23]^2} + \frac{-1}{[12][23][31]^2} + \frac{1}{[12]^2 [23][31]} \right] \ln[31]
\end{aligned}$$

The result for T_{311} is

$$T_{311} = M_4 + K_4$$

Appendix D

In this Appendix we calculate T_{312}

$$T_{312} = -\frac{(31)_\nu}{[31]^2} \int Dy \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \int Dz \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \right) \Pi_{\mu\nu}(z3) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} =$$

$$-\frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \Pi_{\rho\sigma}(z2) \frac{(2y)_\sigma}{[2y]^{2-\epsilon}} \right) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}}$$

We use Eq. (B.1) to present as a sum of four contributions, and the first contribution to T_{312} (with factor -1) is

$$D_1 \equiv \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{2(2y)_\rho}{[2z]^{1-\epsilon} [2y]^{2-\epsilon}} \right) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} =$$

$$\frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{2(2y)_\rho}{[2y]^{2-\epsilon}} \frac{(yz)_\rho}{[yz]^{2-\epsilon}} \frac{1}{[1y]^{1-\epsilon}} =$$

$$-\frac{1}{2(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \left(\partial_\rho^{(y)} \frac{1}{[2y]^{1-\epsilon}} \right) \left(\partial_\rho^{(y)} \frac{1}{[yz]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon}} =$$

$$-\frac{1}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \left[\partial_{(y)}^2 \left(\frac{1}{[2y]^{1-\epsilon} [yz]^{1-\epsilon}} \right) \right. \\ \left. - \left(\partial_{(y)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} - \frac{1}{[2y]^{1-\epsilon}} \left(\partial_{(y)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \right] \frac{1}{[1y]^{1-\epsilon}} =$$

$$-\frac{1}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon} [yz]^{1-\epsilon}} k\delta(1y) \\ + \frac{1}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[1y]^{1-\epsilon} [yz]^{1-\epsilon}} k\delta(2y) \\ + \frac{1}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dy \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} k\delta(yz) =$$

$$-\frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[12]^{1-\epsilon} [1z]^{1-\epsilon}} \\ + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[12]^{1-\epsilon} [2z]^{1-\epsilon}} \\ + \frac{k}{4(1-\epsilon)^2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{1}{[2z]^{1-\epsilon}} \right) \frac{1}{[1z]^{1-\epsilon} [2z]^{1-\epsilon}} =$$

$$-\frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \frac{1}{[12]^{1-\epsilon}} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{2-2\epsilon} [1z]^{1-\epsilon}} \\ + \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{3-2\epsilon} [1z]^{1-\epsilon}} =$$

$$-\frac{k}{4(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(1z)_\mu}{[2z]^{2-2\epsilon} [1z]^{2-\epsilon}} - \frac{k}{2} \frac{(31)_\nu}{[31]^2 [12]} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2 [1z]} \equiv$$

$$\equiv -\frac{k}{8(1-\epsilon)} M_{11} - \frac{k}{2} C_{11}$$

The second contribution to T_{312} is (with factor $-1/(4(1-\epsilon))$)

$$\begin{aligned}
D_2 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \partial_{(2)}^2 \left(\frac{(2z)_\rho}{[2z]^{1-\epsilon} [2y]^{1-\epsilon}} \right) \right) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
&\quad \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon} [2y]^{1-\epsilon}} \right) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
&\quad \frac{1}{2(1-\epsilon)} \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \right) \left(\partial_\rho^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
&\quad \frac{1}{4(1-\epsilon)} \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \left[\partial_{(z)}^2 \left(\frac{(2z)_\mu}{[2z]^{1-\epsilon} [yz]^{1-\epsilon}} \right) \right. \\
&\quad \left. - \left(\partial_{(z)}^2 \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} - \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \right] = \\
&\quad \frac{1}{4} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z][yz]} \\
&\quad + \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z]^2[yz]} \\
&\quad - \frac{k}{4(1-\epsilon)} \partial_{(2)}^2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \frac{(2z)_\mu}{[2z]^{1-\epsilon}} \delta(yz) = \\
&\quad \frac{1}{4} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z][yz]} \\
&\quad + \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z]^2[yz]} - \frac{k}{4} \frac{(31)_\nu}{[31]^2} \partial_{(2)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^2[1z]} \equiv \\
&\quad \equiv \frac{1}{4} B_{41} + B_{51} - \frac{k}{4} B_{42}
\end{aligned}$$

The third contribution to T_{312} is (with factor $1/(4(1-\epsilon))$)

$$\begin{aligned}
D_3 &\equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \left(\partial_{(2)}^2 \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \right) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
&\quad -4(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
&\quad -4(1-\epsilon) \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[2y]^{1-\epsilon}} \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
&\quad -2 \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \left(\partial_\rho^{(z)} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \left(\partial_\rho^{(z)} \frac{1}{[yz]^{1-\epsilon}} \right) = \\
&\quad - \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \left[\partial_{(z)}^2 \left(\frac{(2z)_\mu}{[2z]^{2-\epsilon} [yz]^{1-\epsilon}} \right) \right. \\
&\quad \left. - \left(\partial_{(z)}^2 \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \right) \frac{1}{[yz]^{1-\epsilon}} - \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \left(\partial_{(z)}^2 \frac{1}{[yz]^{1-\epsilon}} \right) \right] = \\
&\quad - \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z]^2[yz]} \\
&\quad - \frac{k}{2(1-\epsilon)} \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \partial_\mu^{(2)} \delta(2z) \frac{1}{[yz]^{1-\epsilon}}
\end{aligned}$$

$$\begin{aligned}
& +k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \frac{(2z)_\mu}{[2z]^{2-\epsilon}} \delta(yz) = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z]^2[yz]} \\
& - k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \frac{1}{[2y]^{1-\epsilon} [1y]^{1-\epsilon}} \delta(2z) \frac{(yz)_\mu}{[yz]^{2-\epsilon}} \\
& + k \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2z]^{3-2\epsilon} [1z]^{1-\epsilon}} = \\
& - \frac{(31)_\nu}{[31]^2} \partial_{(3)}^2 \int Dy Dz \Pi_{\mu\nu}(z3) \frac{(2z)_\mu}{[2y][1y][2z]^2[yz]} - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \Pi_{\mu\nu}(23) \int Dy \frac{(1y)_\mu}{[2y]^{2-2\epsilon} [1y]^{2-\epsilon}} \\
& - \frac{k}{2} \frac{(31)_\nu}{[31]^2} \int Dz \Pi_{\mu\nu}(z3) \frac{(1z)_\mu}{[2z]^{2-2\epsilon} [1z]^{2-\epsilon}} = -B_{61} - \frac{k}{4} M_{10} - \frac{k}{4} M_{11}
\end{aligned}$$

The fourth contribution to T_{312} is zero. Indeed,

$$\begin{aligned}
D_4 & \equiv \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \left(\partial_{(2)}^2 \frac{1}{[2y]^{1-\epsilon}} \right) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \delta(2y) \frac{(yz)_\rho}{[yz]^{2-\epsilon} [1y]^{1-\epsilon}} = \\
& k \frac{(31)_\nu}{[31]^2} \int Dy Dz \Pi_{\mu\nu}(z3) \left(\partial_\mu^{(z)} \frac{(2z)_\rho}{[2z]^{1-\epsilon}} \right) \frac{(2z)_\rho}{[2z]^{2-\epsilon} [12]^{1-\epsilon}} = 0
\end{aligned}$$

The sum of singular integral from D_1 and D_3 contribution is

$$\begin{aligned}
M_5 & \equiv \frac{k}{8(1-\epsilon)} M_{11} - \frac{k}{16(1-\epsilon)} M_{12} - \frac{k}{16(1-\epsilon)} M_{11} = \\
& k \left[\frac{1/16}{[12]^2[23]^2} + \frac{1/16}{[12]^2[31]^2} + \frac{1/16}{[23]^2[31]^2} + \frac{-1/8}{[12][23][31]^2} + \frac{-1/8}{[12][23]^2[31]} + \frac{-1/8}{[12]^2[23][31]} \right] \\
& + k \left[\frac{-1/16}{[12]^2[23]^2} + \frac{1/8}{[12]^2[31]^2} + \frac{-1/16}{[23]^2[31]^2} + \frac{-1/16}{[12][23][31]^2} + \frac{1/8}{[12][23]^2[31]} + \frac{-1/16}{[12][23]^2[31]} \right] \ln[23] \\
& + k \left[\frac{1/16}{[12]^2[23]^2} + \frac{-1/8}{[12]^2[31]^2} + \frac{1/16}{[23]^2[31]^2} + \frac{1/16}{[12][23][31]^2} + \frac{-1/8}{[12][23]^2[31]} + \frac{1/16}{[12][23]^2[31]} \right] \ln[31]
\end{aligned}$$

The sum of finite integral from D_1 , D_2 and D_3

$$\begin{aligned}
K_5 & \equiv \frac{k}{2} C_{11} - \frac{1}{16} B_{41} - \frac{1}{4} B_{51} + \frac{k}{16} B_{42} - \frac{1}{4} B_{61} = \\
& k \left[\frac{-1/8}{[12]^2[23]^2} + \frac{1/8}{[23]^2[31]^2} + \frac{-5/8}{[12][23][31]^2} + \frac{1/8}{[12]^2[23][31]} \right] \\
& + k \left[\frac{1/4}{[12][31]^2} + \frac{-1/4[12]}{[23]^2[31]^2} + \frac{1/4}{[23][31]^2} + \frac{1/4}{[23]^2[31]} \right] J(1, 1, 1) \\
& + k \left[\frac{1/4}{[23]^2[31]^2} + \frac{1/4}{[12][23][31]^2} \right] \ln[12] + k \left[\frac{-1/2}{[12]^2[31]^2} + \frac{-1/4}{[23]^2[31]^2} + \frac{1/4}{[12][23]^2[31]} \right] \ln[23]
\end{aligned}$$

$$\begin{aligned}
& +k \left[\frac{1/2}{[12]^2[31]^2} + \frac{-1/4}{[12][23][31]^2} + \frac{-1/4}{[12][23]^2[31]} \right] \ln[31] \\
& + \left[\frac{1/2}{[12]^2[23]^2} + \frac{-1}{[12][23][31]^2} + \frac{-1/2}{[12][23]^2[31]} + \frac{-1/2}{[12]^2[23][31]} \right] \\
& + \left[\frac{-1}{[12][23]^2} + \frac{-1/2[12]}{[23]^2[31]^2} + \frac{1}{[23][31]^2} + \frac{3/2}{[23]^2[31]} \right] J(1, 1, 1) \\
& + \left[\frac{-1/2}{[12][23][31]^2} + \frac{-1/2}{[12][23]^2[31]} \right] \ln[12] + \left[\frac{-1/2}{[12]^2[23]^2} + \frac{1/2}{[12][23]^2[31]} + \frac{-1/2}{[12]^2[23][31]} \right] \ln[23] \\
& + \left[\frac{1/2}{[12]^2[23]^2} + \frac{1/2}{[12][23][31]^2} + \frac{1/2}{[12]^2[23][31]} \right] \ln[31]
\end{aligned}$$

The result for T_{312} is

$$T_{312} = M_5 + K_5$$

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